CME2300

Financial Engineering

Reader CME2300

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List of symbols

B/C	= profitability index or benefit/cost ratio
C	= Amount of cash flows
C_0	= Call options
E	= Expenditure flow
FV	= Future Value
H_i	= the quantity of the j^{th} item in the Direct costs
I	= receipts, income, revenues
I_t	= Payment on t
K	= Capital
L	= Labor (the number of man hours per year)
M	= Number of additional percentages
N	= Number of items/activities
NPV	= Net Present Value
O_t	= Maintenance costs
P	= Profit
PDF	= Probability Density Function
P_i	= the unit price in the j^{th} item in the direct costs
PV	= Present Value
Q	= Total production (the monetary value of goods produced in one year)
Q_t	= Production per year
S	= Current share value
X	= Strike price
Z	= reliability function
i	= inflation rate
p	= sales price
q	= Quantity
r	= Interest rate, discount rate
r’	= real interest
t	= Time
t_0	= Moment of recognition, t=0
t_i	= The time expenditure flow of a phase end
α, β	= (production function) output elasticity’s of labor and capital, constants defined by available technology
δ	= depreciation
ε	= Efficiency coefficient
σ	= Standard deviation
%j	= the j^{th} addition over the foregoing subtotal

1 Introduction to the course

1.1 Information about the reader

Title: Financial Engineering
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1.2 Information about the course

Module name: Financial Engineering
Module code: CME2300
Period: education period 1
Points: 4 ECTS

1.3 Course outline

This course deals with the finance issues related to the implementation of civil engineering projects. It introduces economic engineering concepts and finance-related topics such as project financing and financial accounting. This course requires the student to study in detail:

- Finance and the firm, covering topics such as sources of finance, cost of finance, financial structures, working capital management and financial accounting.
- Capital Budgeting Decisions and Risk, covering topics such as Capital Budgeting, Political and Social factors, Portfolio Management and Risk Considerations.
- Project finance, covering topics such as international capital markets, stakeholder/actors viewpoints and cash flow modeling of projects including characteristics of typical projects like oil wells, open mining, infrastructure and office buildings.

1.4 Course objectives

The intended learning outcomes of this course are:

- To give students a knowledge of financing and financial implications of civil engineering projects from both a firm and a project perspective, including perspectives from financial involved actors;
- To give students an understanding of the project life-cycle and its impact on and relationship with project finance;
- To learn the students how financial models and techniques should be applied on actual financial problems in corporate finance as well in projects;
- To improve and develop the ability of students to deal with uncertain factors (time, interest) and financial risks.

1.5 Education method

Lectures (4 hours per week). Instructions to prepare for the written exam (multiple choice).

1.6 Expected prior knowledge and prerequisites

There are no formal prerequisites for this module. It will be an advantage when students did follow related courses in economics, probabilistic design, assessment and planning of infrastructure. See paragraph relations to other courses.

1.7 Course material

- Brealey, R. A., Myers, S. C. & Allen, F. (2010), *Principles of corporate finance*, - Tenth Edition New York, McGraw-Hill Global Edition
- This Reader Financial Engineering, June 2013 (CME 2300).
- Other material available on blackboard. Check the latest information on blackboard.

Obligatory chapters of Brealey and Myers:

Part One: Value

Chapter 1: Goals and Governance of the Firm

Chapter 2: How to calculate Present Values

Chapter 3: Valuing Bonds

Chapter 4: The Value of Common Stocks

Chapter 5: Net Present Value and Other Investment Criteria

Chapter 6: Making Investment Decisions with the Net Present Value Rule

Part Two: Risk

Chapter 7: Risk and return

Chapter 8: Portfolio Theory and the Capital Asset Pricing Model

Chapter 9: Risk and the Cost of Capital

Part Three: Best Practices in Capital Budgeting

Chapter 10: Project Analysis

Part Five: Payout Ratio and Capital Structure

Chapter 16: Payout policy

Chapter 17: Does debt policy matter?

Chapter 18: How much should a firm borrow?

Chapter 19: Financing and valuation

Part Six: Options

Chapter 20: Understanding Options

Chapter 21: Valuing Options

Chapter 22: Real Options

Part Seven: Debt Financing

Chapter 23: Credit risk and the value of corporate debt

Chapter 24: The many different kinds of debt

1.8 Relations to other courses

This course is related to other courses at TU Delft, as there are:

- CIE4010: Economics
- CIE4130: Probabilistic Design
- CIE4760: Infrastructure projects: Assessment and Planning
- EPA1233: Economics of Infrastructures
- SPM8000: Project Management
- AR0880: Real estate valuation
- AR1RO46: management & Finance

1.9 For written examination: Assignments

In 2015 the examination will be a written multiple-choice test (40 questions, theory and financial calculations). The examination can be done three times a year in November and January. An extra exam will be hold in August.

1.10 Preparation for plenary lectures

Since the duration of this course is only one period, active mastering of the course material through practice is necessary. Presence during the lectures without reading the course material makes fruitful participation as good as impossible.

2 Financial engineering or computational finance

2.1 Description

Computational finance or **financial engineering** (Wikipedia, 2009b) is a cross-disciplinary field which relies on computational intelligence, mathematical finance, numerical methods and computer simulations to make trading, hedging and investment decisions, as well as facilitating the risk management of those decisions. Utilizing various methods, practitioners of computational finance aim to precisely determine the financial risk that certain financial instruments create.

2.2 History

Generally, individuals who fill positions in computational finance are known as "quants", referring to the quantitative skills necessary to perform the job. Specifically, knowledge of the C++ programming language, as well as of the mathematical subfields of stochastic calculus, multivariate calculus, linear algebra, differential equations, probability theory and statistical inference are often entry level requisites for such a position. C++ has become the dominant language for two main reasons, the computationally intensive nature of many algorithms and the focus on libraries rather than applications.

Computational finance was traditionally populated by PhDs finance, physics and mathematics who moved into the field from more pure, academic backgrounds (either directly from graduate school, or after teaching or research). However, as the actual use of computers has become essential to rapidly carrying out computational finance decisions, a background in computer programming has become useful, and hence many computer programmers enter the field either from Ph.D. programs or from other fields of software engineering. In recent years, advanced computational methods, such as neural network and evolutionary computation have opened new doors in computational finance. Practitioners of computational finance have come from the fields of signal processing and computational fluid dynamics and artificial intelligence.

Masters level degree holders are also increasingly making their presence felt as more terminal programs become available at the leading schools. Today, all full service institutional finance firms employ computational finance professionals in their banking and finance operations (as opposed to being ancillary information technology specialists), while there are many other boutique firms ranging from 20 or fewer employees to several thousand that specialize in quantitative trading alone. JPMorgan Chase & Co. was one of the first firms to create a large derivatives business and employ computational finance (including through the formation of RiskMetrics), while D. E. Shaw & Co. is probably the oldest and largest quant fund (Citadel Investment Group is a major rival).

2.3 Areas of application

Areas where computational finance techniques are employed include:

- Investment banking
- Forecasting
- Risk Management software
- Corporate strategic planning
- Securities trading and financial risk management
- Derivatives trading and risk management
- Investment management
- Pension scheme
- Insurance policy
- Mortgage agreement
- Lottery design
- Islamic banking
- Currency peg
- Gold and commodity valuation

- Collateralized debt obligation
- Credit default swap
- Bargaining
- Market mechanism design

2.4 Major contributors

Some major contributors to computational finance include:

- Fischer Black
- Phelim Boyle
- Emanuel Derman
- Robert Jarrow
- Harry Markowitz *(Nobel Prize 1990)*
- Robert C. Merton
- Stephen Ross
- Myron Scholes *(Nobel Prize 1997)*
- Edward Tsang

2.5 Project finance

Project finance (Wikipedia, 2011c) is the long term financing of infrastructure and industrial projects based upon the projected cash flows of the project rather than the balance sheets of the project sponsors. Usually, a project financing structure involves a number of equity investors, known as sponsors, as well as a syndicate of banks that provide loans to the operation. The loans are most commonly non-recourse or limited recourse loans, which are secured by the project assets and paid entirely from project cash flow, rather than from the general assets or creditworthiness of the project sponsors, a decision in part supported by financial modeling. The financing is typically secured by all of the project assets, including the revenue-producing contracts. Project lenders are given a lien on all of these assets, and are able to assume control of a project if the project company has difficulties complying with the loan terms.

Generally, a special purpose entity/vehicle is created for each project, thereby shielding other assets owned by a project sponsor from the detrimental effects of a project failure. As a special purpose entity, the project company has no assets other than the project. Capital contribution commitments by the owners of the project company are sometimes necessary to ensure that the project is financially sound. Project finance is often more complicated than alternative financing methods. Traditionally, project financing has been most commonly used in the mining, transportation, telecommunication and public utility industries. More recently, particularly in Europe, project-financing principles have been applied to public infrastructure under public–private partnerships (PPP) or, in the UK, Private Finance Initiative (PFI) transactions.

Risk identification and allocation is a key component of project finance. A project may be subject to a number of technical, environmental, economic and political risks, particularly in developing countries and emerging markets. Financial institutions and project sponsors may conclude that the risks inherent in project development and operation are unacceptable (not financeable). To cope with these risks, project sponsors in these industries (such as power plants or railway lines) are generally completed by a number of specialist companies operating in a contractual network with each other that allocates risk in a way that allows financing to take place. "Several long-term contracts such as construction, supply, off-take and concession agreements, along with a variety of joint-ownership structures, are used to align incentives and deter opportunistic behavior by any party involved in the project.” The various patterns of implementation are sometimes referred to as "project delivery methods." The financing of these projects must also be distributed among multiple parties, so as to distribute the risk associated with the project while simultaneously ensuring profits for each party involved.

A riskier or more expensive project may require limited recourse financing secured by a surety from sponsors. A complex project finance structure may incorporate corporate finance, securitization, options, insurance provisions or other types of collateral enhancement to mitigate unallocated risk.

3 Economics and financial engineering

3.1 Introduction

Economics is the social science that analyzes the production, distribution, and consumption of goods and services. Economics can be seen as the science of scarcities. The term economics comes from the Ancient Greek "oikonomia" hence "rules of the house(hold)". Current economic models developed out of the broader field of political economy in the late 19th century, owing to a desire to use an empirical approach more akin to the physical sciences.

Economics aims to explain how economies work and how economic agents interact. Economic analysis is applied throughout society, in business, finance and government, but also in crime, education, the family, health, law, politics, religion, social institutions, war, and science. The expanding domain of economics in the social sciences has been described as economic imperialism.

Common distinctions are drawn between various dimensions of economics. The primary textbook distinction is between microeconomics, which examines the behavior of basic elements in the economy, including individual markets and agents (such as consumers and firms, buyers and sellers), and macroeconomics, which addresses issues affecting an entire economy, including unemployment, inflation, economic growth, and monetary and fiscal policy. Other distinctions include: between positive economics (describing "what is") and normative economics (advocating "what ought to be"); between economic theory and applied economics; between mainstream economics (more "orthodox" dealing with the "rationality-individualism-equilibrium nexus") and heterodox economics (more "radical" dealing with the "institutions-history-social structure nexus"); and between rational and behavioral economics.

It is not always money that plays the most important role in economic analysis. When it is about scarce products (or services), scarce means and uncertainty than economic methods could be used. Time (hours) is scarce and the weather uncertain.

> **Problem statement:** You have to visit Grandma or you can go to the beach, but it could be beautiful or bad weather. What is wise to do?

Four situations may result:

No.	Location	Conditions
1	beach	Rainy
2	beach	Sunny
3	Grandma's place	Rainy
4	Grandma's place	Sunny

Many rules have been developed to decide between going to Grandma or to the beach under uncertain weather conditions. One of these, the **Minimax Regret – technique** can be used. The Minimax Regret approach is to minimize the maximal regret, because it is no fun to sit on a rainy beach or at Grandma's place when the sun is shining bright. The aim of this is to perform as closely as possible to the optimal course. Since the Minimax criterion applied here is to the regret (difference or ratio of the payoffs) rather that to the payoff itself, it is not as pessimistic as the ordinary Minimax approach. These techniques are described in the decision theory. It can be applied in economics (bonds, payoffs), but also in non-monetary cases as the Grandma-Beach case shows us. In economics scarcity and uncertainty are very important issues to deal with as we will see in the next chapters.

3.2 Qualitative and quantitative economic aspects

As the name suggests, quantitative economic aspects deals with annual turnover, profits, total assets, number of employees etc. while the qualitative aspects look at the independence of management and ownership or the non-dominance of market positions.

Figure 3-1 shows three different levels of economic focus, the wide-ranging political science level, the macro-economic level and the micro-economic level. The focus of the political level is in the area of multi-criteria analysis whereas the macro and micro economy focuses on cost benefit analysis and cash flow.

Multi-criteria analysis is described by Nijkamp (1975) as a valuable process as it can be used to reject less valuable projects and to select good alternatives. Multi-criteria analysis can also provide the opportunity to include qualitative factors without transforming them into a monetary dimension, such as environmental consequences. It allows for more possibilities for an objectification of political preferences than cost-benefit analysis, which can be integrated into the multi-criteria analysis.

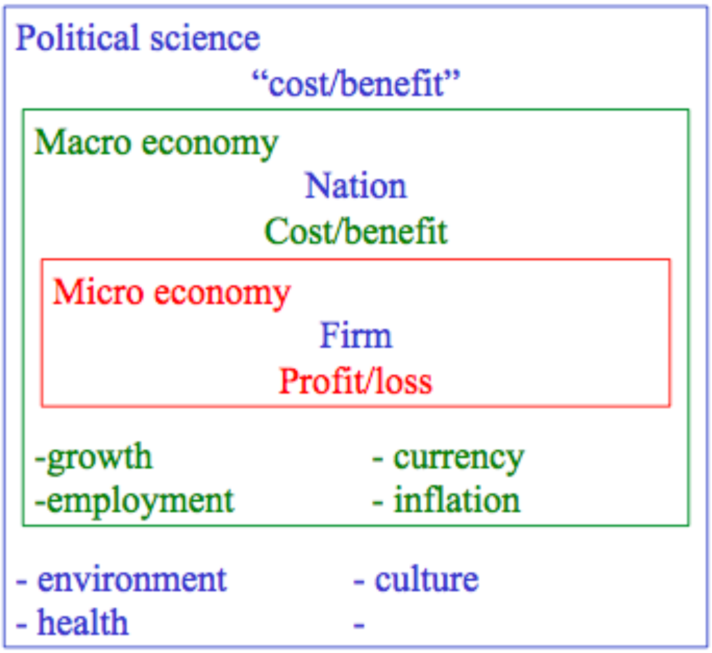


Figure 3-1

Multi-criteria analysis is done by a decision making team which forms objectives and criteria, decides on weights and performance of each criteria and adds risk features into the analysis if necessary. Therefore it relies on the judgment of the decision making team.

By implementing multi-criteria analysis one ensures an open analysis that can be revised if necessary. There are several types of multi-criteria analysis techniques available to use such as the performance matrix, linear additive models, analytical hierarchy process, the outranking method and SMART, to name a few (Nijkamp, 1975).

Cost-benefit analysis (or benefit-cost analysis) is generally more detailed than the multi-criteria analysis. It weighs the probable costs against the probable benefits in order to choose the most profitable and/or valuable option, calculated in the present value. The method is widely used in companies and governments alike to increase efficiency of the firm. As an example, if a product manager does not expect the sales of his product to cover the initial cost of manufacturing and marketing he is unlikely to go ahead with the production. In order for the cost-benefit analysis to be fairly accurate it is important that the less tangible effects, such as loss of reputation in case of a failure is assigned a monetary value. Although the cost-benefit analysis gives a good indication of a project's success there are few pitfalls that need to be avoided, such as heavy reliance on similar previous projects, reliance on the project member's ability to identify risks, negligence of the intangible elements and the unavoidable bias of the project team members (Wikipedia, 2011a). With cost-benefit analysis the costs and benefits are measured relative to a status quo option such as whether or not to clean up a hazardous waste site. By looking at property value for example, it can be determined how much value removing the hazardous waste would add to the homeowners in the area compared to homes that are not affected by the hazardous waste. The cost of the cleanup procedure compared to the added monetary value could be used to determine if the project should be executed or not (Lee Merkhofer Consulting, 2011).

When companies do a cost-benefit analysis for their upcoming projects, one of the aspects that need to be looked into is the expected **cash flow** of the project. As the name suggests, cash flow is the movement of cash into or out of a project/company. Cash inflows can usually be divided into three groups:

1. Financing
2. Operations
3. Investing

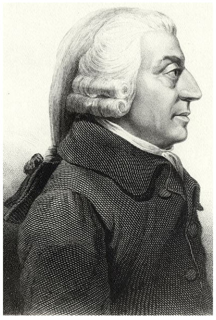
Whereas the cash outflow is either because of:

1. Expenses
2. Investments

Cash flow statement is an accounting statement that shows the amount of money made and used by a company, or a particular project, during a specified period. It is calculated by adding noncash charges, such as depreciation, to net income after taxes and can be used as an indication of a company's financial strength (Investopedia, 2011). Cash flow is considered to be an important or most important financial statistic as cash is the energy that drives business. Companies with big cash flow are popular takeover options as the cash can be used to pay off the debt caused by the purchases (Farlex free dictionary, 2011).

Cash flows can either be used as a record of a previous event or as a forecast for the future projects. It records the company's cash at hand, as comfortable amount ensures that all creditors and personnel can be paid on time as well as makes the company capable of investing back in the business to generate even more cash and profit (Investopedia, 2011).

4 Adam Smith – first economic analysis



Adam Smith (1723 – 1790) was a Scottish moral philosopher and a pioneer of political economics. One of the key figures of the Scottish Enlightenment, Smith is the author of *The Theory of Moral Sentiments* and *An Inquiry into the Nature and Causes of the Wealth of Nations*. The latter, usually abbreviated as *The Wealth of Nations*, is considered his magnum opus and the first modern work of economics. Smith is widely cited as the father of modern economics (Wikipedia, 2010).

Although economics was not yet a term in use when The Wealth of Nations was published, the work did introduce some very important ideas about economics that are discussed here.

4.1 Labour

Division of labour (Smith, 1776, pp. 11-13)
Adam Smith explains the idea of division of labour with the example "from a very trifling manufacture… the trade of a pinmaker":

> A workman not educated to this business, nor acquainted with the use of the machinery employed in it, could scarce, perhaps, with his utmost industry, make one pin in a day, and certainly could not make twenty. But in the way in which this business is now carried on, not only the whole work is a peculiar trade, but it is divided into a number of branches,… One man draws out the wired, another straights it, a third cuts it, a fourth points it, a fifth grinds it at the top for receiving the head; to make the head requires two or three distinct operations; to put it on, is a peculiar business, to whiten the pin is another; it is even a trade by itself to put them into the paper….
> Those ten persons, therefore, could make among them upwards of forty-eight thousand pins in a day.

This division of labour can introduce a proportional increase of the productive powers of labour. The advantages of this division were likely the driving force behind diversification of the trades and industry, and this diversification was greatest for nations with more industry and improvement. Nevertheless, this effect is less strong in agriculture than in manufacturing. Although the lands are better cultivated and the labour is more productive in the rich countries, the poor countries can still rival in the cheapness and goodness of for example its corn, due to their soil, climate and situation of the country.

The increase of quantity of work (the same amount of people producing more) in consequence of the division of work is owing to three circumstances (Smith, 1776, p. 14):

1. The increase of dexterity in every particular workman (Dutch: handigheid)
2. The saving of time lost in passing from one species of work to another
3. The invention of machines

The division of work makes the workmen dependent on each other. Imagine what a variety of work is necessary to produce the tools for each of those workmen. Without the assistance and co-operation of many thousands, the average (Dutch: middelmatige) person in a civilized country could not be provided the easy and simple manner in which he is commonly accommodated (Smith, 1776, p. 20).

Occasion to the Division of Labour (Smith, 1776, pp. 22-24)
This division of labour is the consequence of the propensity of human nature to truck, barter and exchange one thing for another. But, it is not from the benevolence of the butcher, the brewer, or the baker, that we expect our dinner, but from their regard to their own interest: "Give me that what I want, and you shall have this what you want".

The extent of the market (Smith, 1776, pp. 27-32)
As this power of exchanging gives the occasion of the division of labour, so does the limit of this power also limit the extent of the division of labour. The limit of this power = the limit of exchanging = the limit of the market.

There are some sorts of industry, even of the lowest kind, which can be carried on nowhere but in a great town. A porter, for example, can find employment and subsidence in no other place. And a nail maker in the remote and inland parts of the Highlands of Scotland, could make thousand nails a day, three hundred days in a year (three hundred thousand nails in the year). But in such situation it would be impossible to dispose even one thousand nails, one day's work in the year! [p.28]
The improvement of infrastructure extends the market. A broad-wheeled wagon, attended by two men, and drawn by eight horses, in about six weeks' time carries and brings back between London and Edinburgh near four ton weight of goods. Six or eight men by the help of water-carriage, can carry and bring back in the same time the same quantity of goods between London and Edinburgh, as fifty broad-wheeled wagons, attended by hundred men, and drawn by four hundred horses.
Smith shows that the richness of countries depends on the availability of waterborne transport. Think about the Mediterranean, Egypt, China and Bengal.

4.2 Money and price

The use of money
To be able to exchange products, even when you have no product to offer that is interesting for the other, money was 'invented'. First commodities such as salt (in Abyssinia), shells (in India), tobacco (in Virginia) were used, but later all countries chose for metals above other commodities (Smith, 1776, p. 35). These pieces of metal were weighed to determine their value. Also each type of metal has different value. This led to all kind of problems in terms of fraud. Finally, to prevent such abuses, to facilitate exchanges, and thereby encourage all sorts of industry and commerce, it had been found necessary to affix a public stamp upon certain quantities of such particular metals: money (Smith, 1776, p. 37).

Rules about how to exchange products for one another or for money are hard to define. Smith explains it by using the meaning of the word **value**.
There are two meanings of value:

- value in use
- value in exchange

Think about water and diamonds: water is the most valuable thing in use, but can hardly be exchanged for anything and diamonds has hardly any value in use, but is of great value when exchanged for other things (Smith, 1776, p. 41).

The real and nominal price (Smith, 1776, p. 43)
The **real price** of everything, what everything costs to the man who wants to acquire it, is the toil and trouble of acquiring it: what is bought with money or with goods is purchased by labour.
The **nominal price** is the price in terms of money.
The real price of a meal for a student might be more or less an hour of work, a trip by bike and a very small part of the maintenance you do on your bike.
The nominal price is the amount of money showed by the receipt, which may change every day and evolve over time.

That nominal price changes, as the value of gold and silver changes every day, depending on the amount of labour it costs to get it on the market. The discovery of the abundant mines of America reduced, in the sixteenth century, the value of gold and silver in Europe to about a third of what it had been before (Smith, 1776, p. 46).

Concluding, labour alone is the ultimate and real standard by which the value of all commodities at all times can be estimated. This is the real price; money is the nominal price only (Smith, 1776, p. 47).

Natural and market price (Smith, 1776, pp. 78-87)
The **natural price** is what is sufficient to pay the rent of the land, the wages of labour and the profits of the stock employed in raising, preparing and bringing it to the market, according to their natural rates.
This is more or less the lowest price at which the selling party is *likely* to sell its product. This is not necessarily the lowest price at which he will sell the product. A higher price will leave the seller more profit.
The actual price at which a commodity is commonly sold is called the **market price**. This can be higher, lower or exactly the same as the natural price.
The market price is regulated by the quantity that is brought to the market and the demand of those who are willing to pay the natural price (the effectual demand).
When the effectual demand increases, the market price will rise above the natural price. The suppliers of that market will then be careful to conceal this change. If their high profits are known, the market price will soon be reduced to the natural price (or even below it). Nevertheless, secrets of this kind can seldom be kept.
Secrets in manufactures are longer kept than secrets in trade. A dyer who has found the means of producing a particular colour with materials that cost only half the price of those commonly made use of, may, with good management, enjoy the advantage of his discovery as long as he lives, and even leave it as a legacy to his posterity.

A monopoly granted to an individual has the same effect as a secret in trade or manufactures: monopolists will constantly keep the market under stocked, by never supplying the effectual demand, and sell their commodities above the natural price. This monopoly price is generally the highest, which can be got.

4.3 Land and infrastructure

Inequalities of wages and profit
The chance of gain is by every man more or less over-valued, and the chance of loss is by most men under-valued, and by hardly any man valued more than it is worth.
That the chance of gain is naturally over-valued, we may learn from the universal success of **lotteries**. The world will never see a perfectly fair lottery, as the undertaker will have no stake in it. So, the whole gain does not compensate the whole loss, but still there is a large demand for tickets. In order to have a better chance for some great prizes, some people purchase several tickets, and others, small shares in a still greater number. There is not, however, a more certain proposition in mathematics, than that the more tickets you adventure upon, the more likely you are to be a loser.
That the chance of loss is under-valued and hardly valued more than it is worth, we may learn from the very moderate profit of **insurers**. Though many people have made a little money by insurance, only very few have made a great fortune. The common premium is just sufficient to compensate the common losses, the expense of the management and a reasonable profit (what could be drawn from an equal capital in a common trade). A higher premium would lead to less people having insurance, as they value the risk lower than it is.
A great company, having twenty or thirty ships at sea, may have no insurance. The ships insure one another: the premium saved upon them all, may more than compensate the losses that are likely to meet in the common course of chances (Smith, 1776, pp. 149-150).
About the inequalities of wages and profit, Adam Smith writes:

> Apothecaries' profit is become a bye-word, denoting something uncommonly extravagant. This great apparent profit, however, is frequently no more than the reasonable wages of labour. The skill of an apothecary is a much nicer and more delicate matter than that of any artificer whatever; and the trust which is reposed in him is of much greater importance. [...] His reward, therefore, ought to be suitable to his skill and his trust, and it arises generally from the price at which he sells his drugs (Smith, 1776, pp. 154-155).

Laws and regulations can restrict the competition in a market. For example by giving corporations exclusive privileges and limiting the number of people that are allowed to do the work (Smith, 1776, p. 164).

With laws and regulations, it is difficult to prevent meetings of people with the same trade, which leads to conspiracy against the people or contrivance to raise prices. Although a law cannot prevent such meetings, it should not facilitate such assemblies (for example by publicly registering people of the same trade) (Smith, 1776, p. 177).
Adam Smith states that corporations are not necessary for the better government of a trade. He concludes that not the corporations exercise the discipline over workmen, but the customers. The fear of losing the employment (losing the customers) restrains a workman's frauds and corrects his negligence (Smith, 1776, p. 178).

The rate of interest (Smith, 1776, p. 133)
The rate of interest is determined by all kinds of influences. For example, a defect in the law may even raise the interest above what the condition of the country would require. When the law does not enforce the performance of contracts, it puts all borrowers nearly upon the same footing with bankrupts or people of doubtful credit in better regulated countries. High interests in ancient times can be accounted to this uncertainty of recovering money.
When law prohibits interest altogether, the interest will rise due to the difficulty and danger of evading the law.
The high rate of interest among Mahometan nations can be accounted to these two effects.

Rent of land
Adam Smith discusses the importance of infrastructure:

> Good roads, canals, and navigable rivers, by diminishing the expense of carriage, put the remote parts of the country more nearly upon a level with those in the neighborhood of large towns; and on that account they are the greatest of all improvements. They encourage the cultivation of the remote parts, which must always be the most extensive circle of the country. They are advantageous to towns, by breaking down the monopoly of the country in its neighborhood, and they are advantageous to all parts of the country; for though they introduce some rival commodities into the old markets, they open many new markets to its produce (Smith, 1776, p. 202).

The accessibility of land determines partly the value of a piece of land, which can be influenced by infrastructure.
There are more influences. For example, the value of a coal mine depends on situation and fertility. The produce (determined by the fertility of the mine) should pay the expense (determined by the situation: how much labour does it cost to get the coal on the market) (Smith, 1776, p. 225). A metallic mine depends more on its fertility than on its situation. The metals are so valuable that they can bear the cost of long carriage.

Value of infrastructure
Good roads, canals, and navigable rivers, by diminishing the expense of carriage, put the remote parts of the country more nearly upon a level with those in the neighbourhood of the town. They are upon that account the greatest of all improvements. They encourage the cultivation of the remote, which must always be the most extensive circle of the country. They are advantageous to the town, by breaking down the monopoly of the country in its neighbourhood. They are advantageous even to that part of the country. Though they introduce some rival commodities into the old market, they open many new markets to its produce. Monopoly, besides, is a great enemy to good management, which can never be universally established but in consequence of that free and universal competition which forces everybody to have recourse to it for the sake of self-defense. It is not more than fifty years ago, that some of the counties in the neighbourhood of London petitioned the parliament against the extension of the turnpike roads into the remoter counties. Those remoter counties, they pretended, from the cheapness of labour, would be able to sell their grass and corn cheaper in the London market than themselves, and would thereby reduce their rents, and ruin their cultivation. Their rents, however, have risen, and their cultivation has been improved since that time.

Digression of silver
When more abundant mines are discovered, there will be more precious metals brought on the market and their value will decrease. On the other hand, when the wealth of a country increases (the

annual produce of labour increases), a greater quantity of coin is necessary and the value will increase (Smith, 1776, pp. 256-257).

5 Financial evaluation methods

5.1 The time value of money

Time is money, which is true, literally spoken. As a result of the scarcity of means postponing or advancing a payment has a value. This is indicated by the time value of the money. The time value is indicated by an interest rate according the formula

$$FV = I_t * (1+r)^{(t_0 - t)}$$

In which FV = Future value

I_t = Payment on t

t_0 = Moment of recognition, for example 1995

r = Interest, discount rate, time value of money

The time value is easy to understand if it is applied on a payment I, which is, for example, received 5 years ago in 1990. In 1995 the value of this magnitude is:

$$FV = I_{1990} * (1+r)^5$$

On top of this original value, the accumulative interest over 5 years has been added. The reasoning, is also valid for a value, which will be received in the future, say in 2000. The current value of this future payment (PV is present value) is equal to:

$$PV = I_{2000} * (1+r)^{t_0 - 2000} \rightarrow PV = I * (1+r)^{-5}$$

The value of the amount in the past or the future, calculated on the day of today t_o , is indicated by the present value of that amount.

$$PV = I * (1+r)^{(t_0 - t)}$$

5.2 The value of a cash flow

Often there is a flow of payments. The amounts may differ over time. The present value PV of this flow of payments can be calculated by rating all payments separately on the value moment t_0 and sum them at the end.

$$PV = \sum_{t=t_0}^{t_0} I_t * (1+r)^{(t_0 - t)}$$

The mathematic expression is also valid for payments received in the past.

The market value is in this case the amount, which you possess on t_0 , if all receipts are put on a bank account at an interest r.

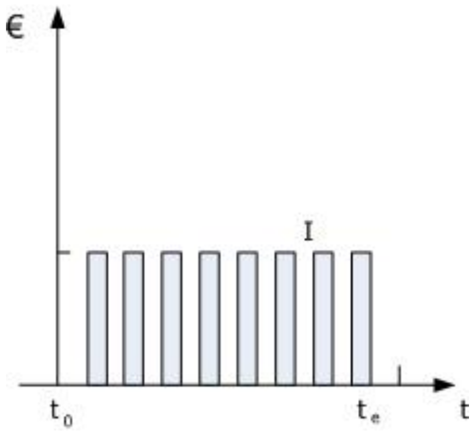


Figure 5-1: Schema of constant cash flow

A special case occurs when the payments are constant and all equal to I.

$$PV(I) = I * \sum_{t=t_0}^{t_0} (1+r)^{(t_0-t)}$$

The market value can now be determined as the sum of a geometric series where t_e is end date of the calculated period.

$$PV(I) = \frac{I}{r} * (1-(1+r)^{(t_0-t_e)})$$

$$PV(I) = I * \frac{1}{r} * (1 - \frac{1}{(1+r)^{(t_e-t_0)}})$$

The market value is found by multiplying the flow I with a factor. This market value factor is often given in handbooks in the form of a table.

$$PV(I) = I * PV\_factor$$

$$PV\_factor = \frac{1}{r}\left[1 - \frac{1}{(1+r)^{(t_e-t_0)}}\right] = f\{r, (t_e - t_0)\}$$

(see table, next page)

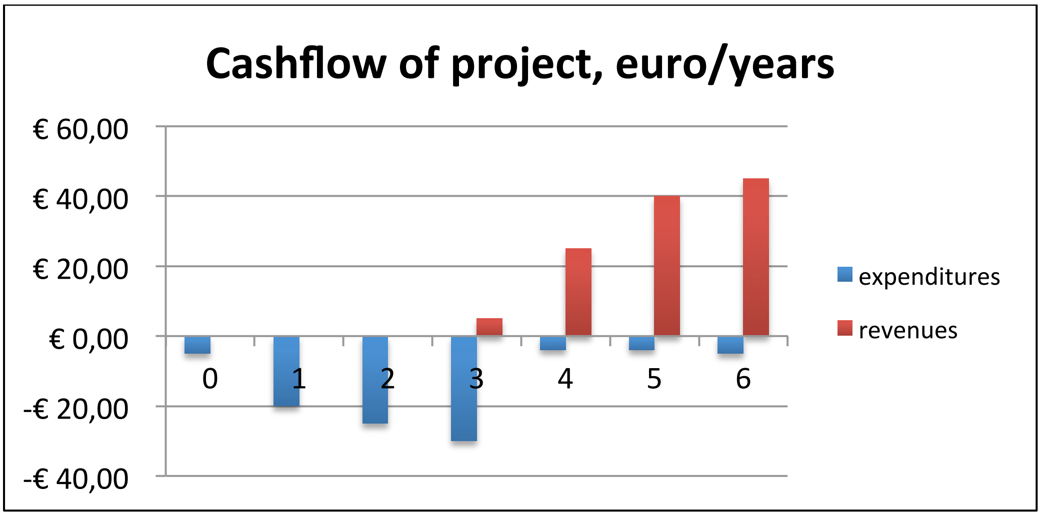
n r	1	1.25	1.5	2	2.5	3	4	5	6
1	0.9901	0.9877	0.9852	0.9804	0.9756	0.9709	0.9615	0.9524	0.9434
2	1.9704	1.9631	1.9559	1.9416	1.9274	1.9135	1.8861	1.8594	1.8334
3	2.9410	2.9265	2.9122	2.8839	2.8560	2.8286	2.7751	2.7232	2.6730
4	3.9020	3.8781	3.8544	3.8077	3.7620	3.7171	3.6299	3.5460	3.4651
5	4.8534	4.8178	4.7826	4.7135	4.6458	4.5797	4.4518	4.3295	4.2124
6	5.7955	5.7460	5.6972	5.6014	5.5081	5.4172	5.2421	5.0757	4.9173
7	6.7282	6.6627	6.5982	6.4720	6.3494	6.2303	6.0021	5.7864	5.5824
8	7.6517	7.5681	7.4859	7.3255	7.1701	7.0197	6.7327	6.4632	6.2098
9	8.5660	8.4623	8.3605	8.1622	7.9709	7.7861	7.4353	7.1078	6.8017
10	9.4713	9.3455	9.2222	8.9826	8.7521	8.5302	8.1109	7.7217	7.3601
11	10.367	10.217	10.0711	9.7868	9.5142	9.2526	8.7605	8.3064	7.8869
12	11.255	11.079	10.907	10.575	10.2578	9.9540	9.3851	8.8633	8.3838
13	12.1337	11.9302	11.7315	11.3484	10.9832	10.6350	9.9856	9.3936	8.8527
14	13.0037	12.7706	12.5434	12.1062	11.6909	11.2961	10.5631	9.8986	9.2950
15	13.8651	13.6005	13.3432	12.8493	12.3814	11.9379	11.1184	10.3797	9.7122
16	14.7179	14.4203	14.1313	13.5777	13.0550	12.5611	11.6523	10.8378	10.1059
17	15.5623	15.2299	14.9076	14.2919	13.7122	13.1661	12.1657	11.2741	10.4773
18	16.3983	16.0295	15.6726	14.9920	14.3534	13.7535	12.6593	11.6896	10.8276
19	17.2260	16.8193	16.4262	15.6785	14.9789	14.3238	13.1339	12.0853	11.1581
20	18.0456	17.5993	17.1686	16.3514	15.5892	14.8775	13.5903	12.4622	11.4699
21	18.8570	18.3697	17.9001	17.0112	16.1845	15.4150	14.0292	12.8212	11.7641
22	19.6604	19.1306	18.6208	17.6580	16.7654	15.9369	14.4511	13.1630	12.0416
23	20.4558	19.8820	19.3309	18.2922	17.3321	16.4436	14.8568	13.4886	12.3034
24	21.2434	20.6242	20.0304	18.9139	17.8850	16.9355	15.2470	13.7986	12.5504
25	22.0232	21.3573	20.7196	19.5235	18.4244	17.4131	15.6221	14.0939	12.7834
26	22.7952	22.0813	21.3986	20.1210	18.9506	17.8768	15.9828	14.3752	13.0032
27	23.5596	22.7963	22.0676	20.7069	19.4640	18.3270	16.3296	14.6430	13.2105
28	24.3164	23.5025	22.7267	21.2813	19.9649	18.7641	16.6631	14.8981	13.4062
29	25.0658	24.2000	23.3761	21.8444	20.4535	19.1885	16.9837	15.1411	13.5907
30	25.8077	24.8889	24.0158	22.3965	20.9303	19.6004	17.2920	15.3725	13.7648
31	26.5423	25.5693	24.6461	22.9377	21.3954	20.0004	17.5885	15.5928	13.9291
32	27.2696	26.2413	25.2671	23.4683	21.8492	20.3888	17.8736	15.8027	14.0840
33	27.9897	26.9050	25.8790	23.9886	22.2919	20.7658	18.1476	16.0025	14.2302
34	28.7027	27.5605	26.4817	24.4986	22.7238	21.1318	18.4112	16.1929	14.3681
35	29.4086	28.2079	27.0756	24.9986	23.1452	21.4872	18.6646	16.3742	14.4982
36	30.1075	28.8473	27.6607	25.4888	23.5563	21.8323	18.9083	16.5469	14.6210
37	30.7995	29.4788	28.2371	25.9695	23.9573	22.1672	19.1426	16.7113	14.7368
38	31.4847	30.1025	28.8051	26.4406	24.3486	22.4925	19.3679	16.8679	14.8460
39	32.1630	30.7185	29.3646	26.9026	24.7303	22.8082	19.5845	17.0170	14.9491
40	32.8347	31.3269	29.9158	27.3555	25.1028	23.1148	19.7928	17.1591	15.0463
41	33.4997	31.9278	30.4590	27.7995	25.4661	23.4124	19.9931	17.2944	15.1380
42	34.1581	32.5213	30.9941	28.2348	25.8206	23.7014	20.1856	17.4232	15.2245
43	34.8100	33.1075	31.5212	28.6616	26.1664	23.9819	20.3708	17.5459	15.3062
44	35.4555	33.6864	32.0406	29.0800	26.5038	24.2543	20.5488	17.6628	15.3832
45	36.0945	34.2582	32.5523	29.4902	26.8330	24.5187	20.7200	17.7741	15.4558
46	36.7272	34.8229	33.0565	29.8923	27.1542	24.7754	20.8847	17.8801	15.5244
47	37.3537	35.3806	33.5532	30.2866	27.4675	25.0247	21.0429	17.9810	15.5890
48	37.9740	35.9315	34.0426	30.6731	27.7732	25.2667	21.1951	18.0772	15.6500
49	38.5881	36.4755	34.5247	31.0521	28.0714	25.5017	21.3415	18.1687	15.7076
50	39.1961	37.0129	34.9997	31.4236	28.3623	25.7298	21.4822	18.2559	15.7619

Table 5-1: Annuity table: See also (Brealey, et al., 2008 :A-3)

5.3 Project evaluation methods

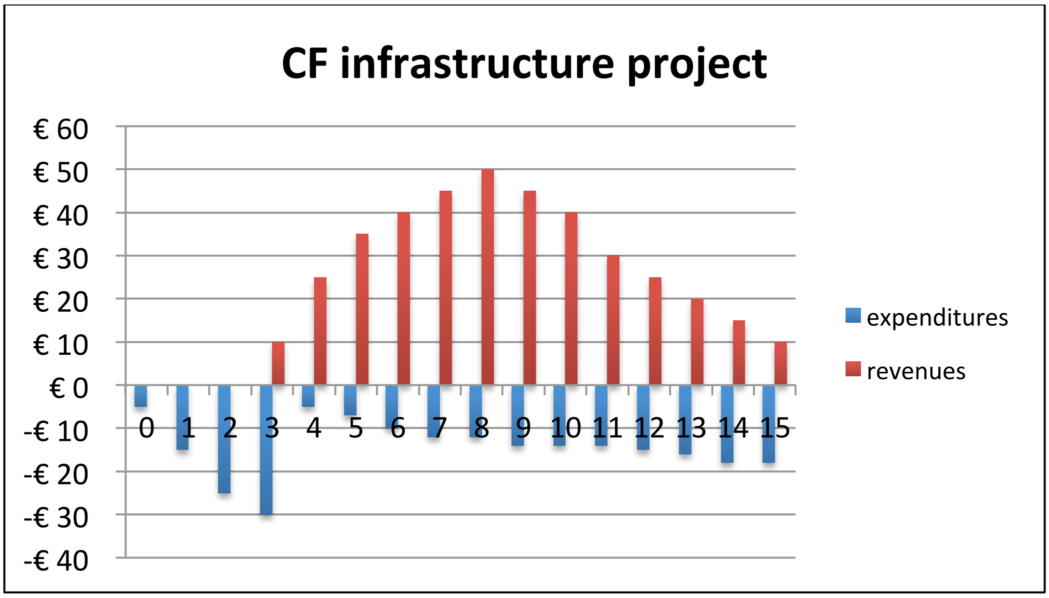
For an economist a project is built up of a series of cash outflows and the followed series of cash inflows.

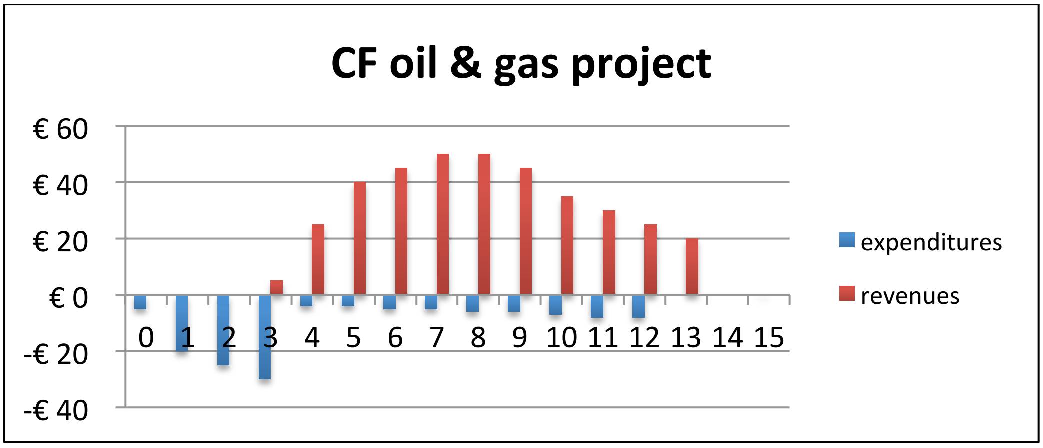
The series of cash outflows in the first years is formed by investments, also called capital expenditures (CAPEX). The inflows are the net revenues of the project. That is the difference between the revenues and the costs of running, the operational expenditures (OPEX).

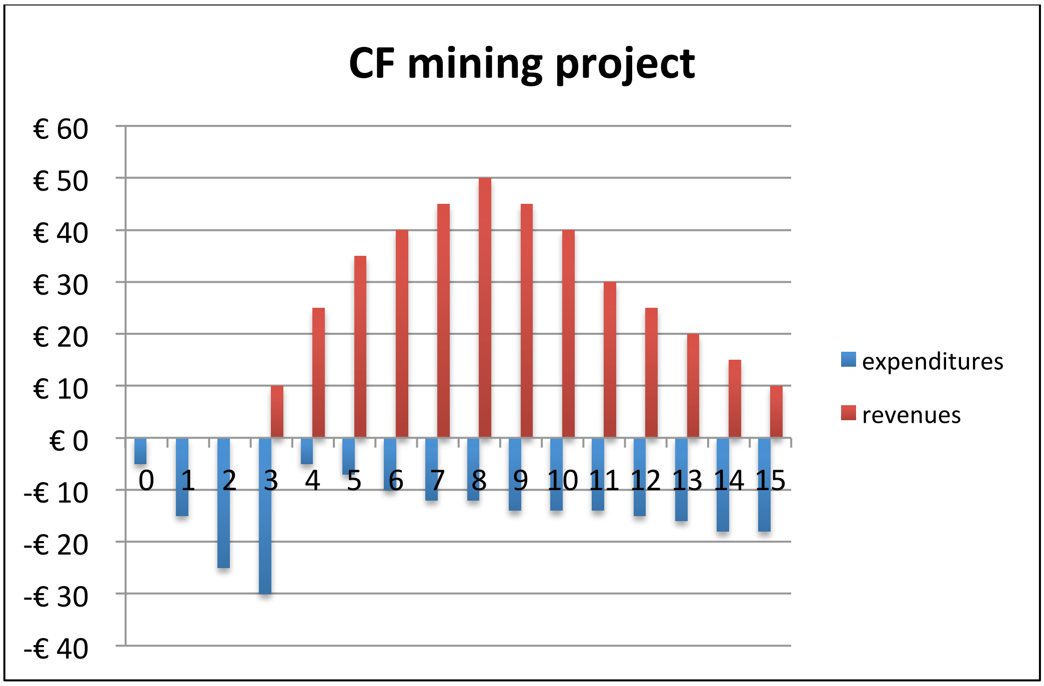


CAPEX: investments, expenditures in the first three years
OPEX: operational expenditures in the years 3 until 6.

The projects could be of a certain type. So, projects in the infrastructure, oil-industry or mining do have typical cash flow patterns.







There are a couple of methods to evaluate the economic importance of such a project.

1. The net value method sums the cash flow over the lifetime cycle of the project. The net value should be positive

$$\sum_{t=t_0}^{t_e} I_t > 0$$

2. The payback period is the period until the net value of the project is exactly zero.

$$\sum_{t=0}^{t_{payback}} I_t = 0 \rightarrow t_{payback} = \ldots$$

The two mentioned methods are not very accurate because the time value of the money is not into play.

3. The net market value method sums the cash flows over the life time cycle of the project with due regard for the time value of the money. The net market value should be positive.

$$NPV = \sum_{t=t_0}^{t_e} I_t * (1+r)^{(t_0-t)} > 0$$

The net present value is not invariant for the scale of the project. Therefore, one often chooses for the profitability index.

4. The profitability index is defined as the ratio between the market value and the net receipts in the production phase and the market value of the expenditure in the building- or investment phase.

$$\frac{B}{C} = -\frac{\sum_{t=t_i}^{t_e} I_t * (1+r)^{(t_0-t)}}{\sum_{t=t_0}^{t_i} I_t * (1+r)^{(t_0-t)}} \quad \begin{matrix} I_t < 0 & t < t_i \\ I_t > 0 & t > t_i \end{matrix}$$

Another advantage of the profitability index is that the moment of appraisal (t_0) has no influence.

5. In the literature, one can also find the internal rate of return method. The criterion is the rate of return at which the NPV of the project is zero.

$$NPV = \sum_{t=t_0}^{t} I_t * (1+r)^{(t_0-t)} = 0 \rightarrow r = r_{\text{intern}}$$

In general, one prefers the net market value method or the profitability index in evaluating the project.

5.4 Project evaluation

A project can financially be seen as a number of expenditures (cash outflow) and a number of receipts (cash inflow).

The expenditure flows can be rubricated as part of the investment (e.g. concrete work, ground work and mechanical parts etc.) capex or as kind of operational expenditure (e.g. wages, material, taxes etc.).
On a certain moment, the project flows from the construction phase into the production phase. The character of the expenditure flow changes from the investment costs (capex) into the exploitation costs (opex) and a notable revenue flow is initiated.

For the benefit of the investment, the expenditure flows can be summed to the total expenditure flow in the building phase, which ends at t_i.

The revenue flows and the exploitation flows can also be rubricated to the source (company income, exploitation costs, allowances etc.) and summed to the total net cash flow in the production phase.

Subsequently, the market value of both the total expenditure flow *E* during the construction phase as well as the net receipts flow *I* during the production phase is determined.

$$PV(E) = \sum_{t=t_0}^{t_i} E_t * (1+r)^{(t_0-t)}$$

$$PV(I) = \sum_{t=t_i}^{t_e} I_t * (1+r)^{(t_0-t)}$$

The net market value of the project is the sum of the market value of the receipts *I* and the expenditures *E.*

$$NPV = PV(I) + PV(E) > 0$$

The profitability index or benefit/cost ratio

$$\frac{B}{C} = -\frac{PV(I)}{PV(E)} > 1.0$$

For a profitable project, the net present value should be positive and the profitability index should be larger than 1.0.

5.5 Sunk costs (Dutch: gedane zaken)

Apart from the question if a project is profitable, one should also investigate how much capital should be invested in the project. In principle, this is the sum of the investment costs, raised by the interest during construction.

If the project has losses during the first years of operation, the capital demands will still increase after the production is started.

The definition of the capital possession as a function of the project phase t* is relatively simple. One considers the cash flow, which is realized up till the moment t*.

$$\text{PV (sunk costs)} = \sum_{t=t_0}^{t^*} C_t * (1+r)^{(t^*-t)}$$

Each year, the expenditures are summed, after being provided with the accumulated interest rate. This is the capital possession as it has developed at t*.

The market value of the future cash flow over the period of t* through te will be added to the expression of the net market value definition at t*.

$$NPV = \sum_{t=t_0}^{t_e} C_t * (1+r)^{(t^*-t)}$$

The difference is the upper limit of the summation and the moment of evaluating t* instead of t_0.

The present value of the sunk costs (d. gedane zaken), or the capital usage, should be positive at the end of the project at t_e. It is the final profit of the project.

$$\text{PV (capital usage)} = \sum_{t=t_0}^{t_e} C_t * (1+r)^{(t_e-t)} \geq 0$$

$$\text{PV (capital usage)} = \sum_{t=t_0}^{t_i} C_t * (1+r)^{(t_e-t)} + \sum_{t=t_i}^{t_e} C_t * (1+r)^{(t_e-t)} \geq 0$$

Apart from one factor, this final profit is equal to the NPV at the start of the project. The factor gives the time value difference between t_0 and t_e:

$$\frac{(1+r)^{t_e}}{(1+r)^{t_0}} * NCW = \frac{(1+r)^{t_e}}{(1+r)^{t_0}} * \left\{ \sum_{t=t_0}^{t_i} C_t * (1+r)^{(t_0-t)} + \sum_{t=t_i}^{t_e} C_t * (1+r)^{(t_0-t)} \right\} \geq 0$$

It follows that the B/C ratio should also be ≥ 1

$$\frac{B}{C} = \frac{\sum_{t=t_i}^{t_e} C_t * (1+r)^{(t_0-t)}}{\sum_{t=t_0}^{t_i} C_t * (1+r)^{(t_0-t)}} \geq 1$$

5.6 Inflation in financial calculations

Cash flow series are expressed in monetary units (dollars, euros or yen). Cash flow does respond to real transactions. That means that changes in the value of valuta do have consequences for cash flow even if the real transactions do no change. One can say that the yardstick is shrinking.

An amount from a real transaction on time t can be expressed in values at t_0 or t_n. When the inflation rate *i* is constant, than the expression will be:

$$C_t = C_t^0 * (1+i)^{(t-t_0)}$$

When the inflation rate *i* varies in time,

$$C_t = C_t^0 * \prod_{t=t_0}^{t}(1+i_t)$$

In case of this kind of transformations, cash flows will have the value for the moment the real transaction occurs. This is of great importance because loan contracts are mostly in nominal amounts of money. The payments are constant in value in euros at that very moment.

When inflation occurs the cash inflow will increase in nominal amounts of money.

The PV of a cash flow like that, becomes:

$$PV = \sum_{t=t_0}^{t} C_t * (1+r)^{t_0-t} * (1+i)^{t-t_0}$$

$$PV = \sum_{t=t_0}^{t} C_t * \left[\frac{(1+r)}{(1+i)}\right]^{t_0-t}$$

The inflation factor can be written as a binomial series as follows:

$$\frac{1}{(1+i)} = 1 - i + i^2 - i^3 + \ldots$$

Than the next expression for the series will follow:

$$\frac{(1+r)}{(1+i)} = (1+r)(1 - i + i^2 - i^3 + \ldots)$$

$$= 1 - i + i^2 - i^3 + r - ri + ri^2 - ri^3 \ldots$$

$$= 1 + r - i - ri + i^2 \ldots$$

The error be estimated as *"r times i"*, that's true for small values of *i* and *r*.

$$So, \frac{(1+r)}{(1+i)} \approx 1 + r - i \qquad if\ r \ll 1\ and\ i \ll 1$$

Under this condition the PV of a constant cash flow can be approximately:

$$PV = \sum_{t=t_0}^{t_e} C_t * (1 + r - i)^{(t_0 - t)}$$

This approach leads to the expression, real interest: *r'=r-i*
Which is often used, but still is an estimate and is only valid for low interest and inflation rates.

5.7 Depreciation and interest

In previous paragraphs we described appraisal methods based on discounted cash flows. All amounts were discounted to a certain period in time and were added together. Other methods want to calculate the costs of a certain "thing" (such as a new product, a machine etc.) (Horngren, A., Datar, & Foster, 2002: p33). To guide their decision, managers often want to know how much that "thing" costs. We call that "thing" a cost object, which is everything for which a separate measurement of costs is desired. In such a costing system accounts costs in two basic stages:

It accumulates costs by some 'natural' classification such as materials, labor, fuel …
It assigns these costs to cost objects.

Cost assignment is a general term that encompasses both (a) tracing accumulated costs to a cost object and (b) allocating accumulated costs to a cost object.

The cost object could be a period of time (year or hour) or the product that initiates the project. By comparing costs and revenues will give more information about the profitability of the project. These methods do we call 'cost accounting'.

For costs that are regular in time we can make an easy calculation.

E.g. costs of yearly maintenance are assigned to cost objects of that year:

$$tariff_o = \frac{O_t}{Q_t}$$ in euro per unit, that is €/u

in which: O_t is maintenance costs €/year
Q_t is production per year u/year

For variable costs that fluctuate with production like material- and labor costs the calculation is:

$$tariff_v = \frac{Q_t * tariff_{material+wages}}{Q_t}$$

$$tariff_v = tariff_{material+wages}$$

Problems will arise when we want to assign the investments to the cost objects over the life cycle of the project.
Depreciation is a method to fix that problem. There are many schemas for that. The linear schema gives the following expression:

$$D_t = \frac{investment}{lifetime} = \frac{investment}{t_e - t_i}$$ in €/year

For assigning the costs to a product:

$$tariff_D = \frac{investment}{(t_e - t_i) * Q_t}$$ €/unit

in which, *investment = construction cost + construction interest*

The investment demands for capital, so interest should be paid. The interest cost per year decreases every year, because the residual value is diminishing.

$$interest_t = r * investment * (1 - \frac{t - t_1}{t_e - t_i}) \quad \text{€/year}$$

Interest costs are assigned to the cost object.

$$tariff_R = \frac{interest}{Q_t} \quad \text{€/unit}$$

Total cost of the cost object (or product) is:

$$total\ cost = tariff_o + tariff_{M+L} + tariff_D + tariff_R$$

The total cost will decrease the next years because the interest part is decreasing, as can be seen in the next table.

Investment = € 1000
Life time = 10 year
Residual value = 0
Interest = 5%

year	Depreciation	Capital	Interest	Depreciation + Interest
1	100	1000	50	150
2	100	900	45	145
3	100	800	40	140
4	100	700	35	135
5	100	600	30	130
6	100	500	25	125
..	...	...		...
10	100	100	5	105

Table 5-2: Straight-line depreciation

That could be inconvenient. Two methods give 'better' results.

Interest on an average each year, that is € 25 (see following Table)

The annual interest payment is calculated by using the simple formula

$$interest = r * \frac{investment}{2}$$

In this example: interest=0,05*(1000/2)=25 per year.

year	Depreciation	Interest	Depr. + Interest	Annuity
1	100	25	125	129,5
2	100	25	125	129,5
3	100	25	125	129,5
4	100	25	125	129,5
5	100	25	125	129,5
6	100	25	125	129,5
..	…			
10	100	25	125	129,5

Table 5-3: Expenses based on constant expenses and annuity

Annuity: every payments are the same

The annuity can be calculated by using formula:

$$investment = annuity * \frac{1-(1+r)^{(t_i-t_e)}}{r}$$

The other way around:

$$annuity = investment * \frac{r}{1-(1+r)^{(t_i-t_e)}}$$

The first method A is quite simple, easy to use, but doesn't make use of the time-value-of-money. When the nominal interest is high and the life time is long, then the error will be considerable.

5.8 Cost price evaluation of projects

Because the assessment of the profitability of projects is often based on the difference between sales price and cost price, it seems good to look at the relationship between these approaches and the previously presented methods. The use of both approaches at the same time is often confusing.

The profitability of a project is determined by comparing the sales revenues to the total cost price.

The difference, the profit per unit, should be positive.

$$profit = sales\ price - cost\ price \geq 0$$

The profit margin can be derived as follows:

$$\frac{sales\ price}{cost\ price} = 1 + profit\ margin\ \geq 1{,}0$$

The last expression is our favorite, because it is a relative expression, the level of prices is of no importance. The question is, is there a relationship between this expression and the previously mentioned profitability index (B/C)?

The ratio can be written as:

$$\frac{sales\ price}{cost\ price} = \frac{sales\ price}{tariff_O + tariff_{M+L} + tariff_A + tariff_R}$$

For simplicity reasons, we assume that the 'depreciation' part and 'interest' part are equal to the 'annuity' part.

$$tariff_D + tariff_R = tariff_{Ann}$$

The profitability index is constituted of PV of the cash inflow (I) and PV of the initial investment (O).

$$\frac{B}{C} = -\frac{PV(I)}{PV(O)}$$

The relation between prices and cash flow (I and O) per year is drawn by the quantity Q that is produced and sold. The gross amount of income is determined by

$$Q * sales\ price$$

The net cash flow is accounted by subtracting the costs of maintenance and materials.

$$I = Q * (sales\ price - \ tariff_{maintenance} - tariff_{material+wages})$$

The present value of this investment, when it is assumed to be constant over life time, will be

$$PV(I) = \frac{1-(1+r)^{(t_i - t_e)}}{r} * I = PF\_factor * I$$

In the same way we can determine the PV of the investment as derived from depreciation and interest.

$$PV(U) = \frac{1-(1+r)^{(t_i - t_e)}}{r} * Q + tariff_{Ann} = PF\_factor * Q * tariff_{Ann}$$

A small difference with the previously derived methods of project appraisal is that the cash flow is discounted to the end t_i of the construction period instead of t_0. But as mentioned before is the profitability index is that the moment of appraisal (t_n) has no influence on B/C ratio.

The cost-benefit ratio can be derived from the expression

$$\frac{B}{C} = \frac{PF\_factor * Q * (sales\ price - tariff_{Maintenance} - tariff_{Materials\ and\ wages})}{PF\_factor * Q * tariff_{Ann}}$$

Simplifying this expression leads to

$$\frac{B}{C} = \frac{sales\ price - tariff_{Maintenance} - tariff_{Materials\ and\ wages}}{tariff_{Ann}}$$

For an example, see chapter 7, especially Westerschelde Bridge.

5.9 Some remarks

A profitability model will assess the financial gain on the use of capital during a period of operations. Profitability models as a general class of models have advantages and disadvantages that include (Meredith & Mantel, 2009):

1) *Advantages*:
    a) They are simple to use and to understand.
    b) Relevant data are available from the accounting system.
    c) Business decision-makers are familiar with the output formats.
    d) The decision-maker, with few exceptions, will assume that the output of the model is absolute regarding a go or no go selection option. In this context, absolute indicates a lack of ambiguity regarding the appropriate decision.
    e) Some profitability models can account for the amount, timing and risk of a project's cash flows.
2) *Disadvantages*:
    a) Except for risk factors, the models ignore other nonmonetary factors.
    b) Some of the profitability models do not evaluate the timing of cash flows.
    c) Present-value models have a short-term bias that tends to ignore long-run opportunities.
    d) Payback models ignore cash flows beyond the time needed to recover the original investment.
    e) Algebraically, the IRR can generate multiple solutions. However, modelers can address this problem by using graphical approximation techniques capable of producing a satisfactory level of accuracy for most planning purposes.
    f) All models in this class are sensitive to data input errors, especially during the early periods of the project's planning horizon.
    g) Because all discounting models are nonlinear, decision-makers are seldom able to recognize the impact of errors in and changes to the values of parameters used in the models.
    h) The definition of cash flow for a project is subject to some degree of ambiguity. Modelers will not always be able to apply the concept consistently when evaluating the financial aspects of a project.

6 Banks

6.1 How a bank is financed

The most basic way a bank finances itself is by the customers' deposits. The bank gets a deposit from one client and lends that same money to another client. The bank then charges higher interest on the loans than it pays to the depositors and keeps the difference. The bank cannot lend the whole amount received by the depositors but has to keep a reserve set by the authorities to be able to pay back the depositing clients when needed. One can think of a bank with 10% reserve requirement, that receives a €100 deposit. The bank then lends out €90 to another client who uses the money for a house, car, services, goods etc. putting the money back into the economy. This €90 usually ends up deposited in another bank that can repeat the lending process of the first bank by loaning €81 (€90-10%). Other sources of income are charges connected to checking accounts, ATM access, overdraft protection etc.

6.2 CDO's - AAA's

The Collateralized Debt Obligations, or CDO's, are securities that derive their payments and worth from their underlying assets, mortgages for example. The CDO's are cut into slices, or tranches, and sold to investors. Each tranche has a different risk class with the senior tranche, in many cases rated AAA, considered the safest slice. Interest and principal payments are determined according to seniority with the junior tranches having the highest interest and/or bearing the lowest price to compensate for the highest risk. In between these two classes is the mezzanine class which carries higher interest than the senior tranche but also higher risk.

In general, the CDO's are held in a Special Purpose Vehicle (SPV) that issues bonds to investors. These bonds, or tranches carry different risks and therefore carry different values. The SPV uses the money from the bonds to purchase a portfolio of underlying assets, mortgages from the mortgage issuing bank for example. Once the mortgage borrowers start paying back the money goes into the SPV who then spreads it to the bond holders, with the senior tranche being served first, then the mezzanine tranche and what is left goes to the junior tranche.

6.3 Bailouts

With the turmoil that shook the financial world in the late 00's various national governments chose to save the local banks from insolvency. As the banks were getting short on equity they started lending out less and less money in order to prepare themselves to pay off their own debt maturing in the near future. This lead to an increasing shortage of money available for the public to invest for, e.g. as for farmers to buy seeds for next year's crop, as well as the banks were no longer lending to each other. Previously, the financial institutions were lending to one another which lead to systemic risk which means that if one bank cannot pay its loan, it has consequences throughout the system. The US is a good example to look into for this matter. The US Central Bank started loaning other banks vast amount of money with collateral in their CDO's in order to stimulate lending to the public. This, however, did not work as banks were still concerned about their own obligations and therefore did not lend out this newly acquired funds. As the banks were now able to pay off their own maturing debt, the equity went to their lenders. This chain of events has been described as moral hazard as the risk falls on taxpayers, not the banks lenders who had obtained the benefits of the risk previously.

6.4 GDP & GNP

Gross Domestic Product (GDP) is often used as a sign for country's living standard. It refers to the total market value of all final goods and services produced in a **country** in a given period. It is equal to private consumption, gross investment, plus exports minus imports.

Gross National Product (GNP) is the total market value of all final goods and services produced in a **nation** in a given period. GNP measures the value of goods and services that the country's citizens produced, regardless of their location.
The difference between GDP and GNP according to Wikipedia is: "GDP can be contrasted with gross national product (GNP) or gross national income (GNI). The difference is that GDP defines its scope according to location, while GNP defines its scope according to ownership. In a global context, world GDP and world GNP are therefore equivalent terms.
GDP is product produced within a country's borders; GNP is product produced by enterprises owned by a country's citizens. The two would be the same if all of the productive enterprises in a country were owned by its own citizens, and those citizens did not own productive enterprises in any other countries. In practices, however, foreign ownership makes GDP and GNP non-identical. Production within a country's borders, but by an enterprise owned by somebody outside the country, counts as part of its GDP but not its GNP; on the other hand, production by an enterprise located outside the country, but owned by one of its citizens, counts as part of its GNP but not its GDP.
To take the United States as an example, the U.S.'s GNP is the value of output produced by American-owned firms, regardless of where the firms are located. Similarly, if a country becomes increasingly in debt, and spends large amounts of income servicing this debt this will be reflected in a decreased GNI but not a decreased GDP. Similarly, if a country sells off its resources to entities outside their country this will also be reflected over time in decreased GNI, but not decreased GDP. This would make the use of GDP more attractive for politicians in countries with increasing national debt and decreasing assets" (Wikipedia, 2011b).

7 Case studies

7.1 Introduction

In this chapter we will highlight some projects in which some aspects of Financial Engineering are of special interest. Such as:

- Time value of money
- Project appraisal
- The influence of delays in construction time, inflation and economic growth

The projects will be:

- The sand motor (new policy)
- Westerschelde Bridge
- Channel Tunnel between France and the UK

7.2 The Sand Motor project

The Sand Motor project is suggested to form a coast in the Netherlands by using natural resources and elements. "Let's nourish the beach", they say. The Sand motor will be a large amount of sand before the coast of Hook of Holland. In time a natural coast will be formed for safety against floods by wind, waves and sea currents. The Sand Motor will look like a hook, which is for 1,5 km into the sea (see artist impression below).

Figure 7-1: Artist impression of Sand Motor project (Projectbureau Pilot Zandmotor Delflandse Kust, 2009)

The promise is: *Slowly but surely the forces of nature form a concave beach and natural dunes are formed.* The expectations are the following phases, which will tend to Figure 7-4, the future situation (30 years from now) is a natural beach. The project will start with an initial nourishment at two different locations will be necessary. Yearly maintenance is necessary.

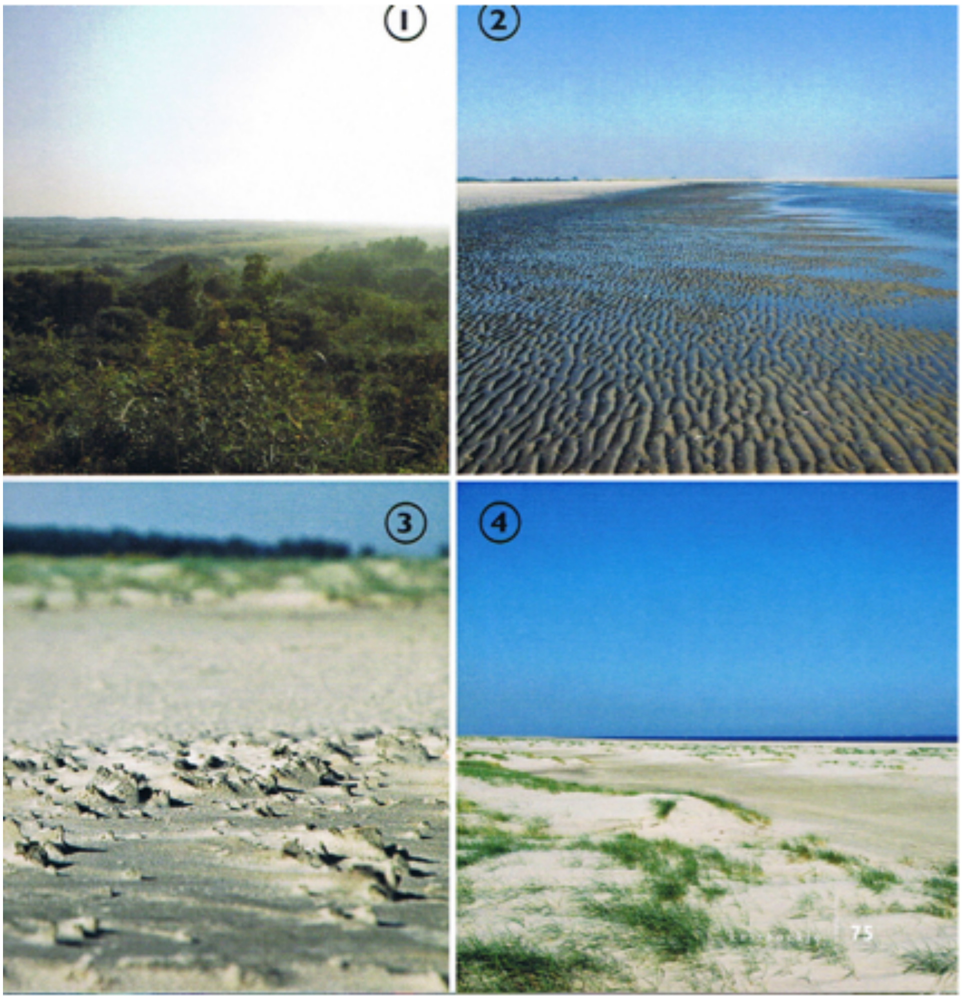


Figure 7-2: The results of the sand motor in 4 stages

Before the coast developments for recreation and nature will be realized in the future.

But what will it cost? Before making financial calculations data is needed. First about the location.

The project is located near The Hague and Rotterdam in the neighborhood of Hook van Holland as can be seen on the next map.

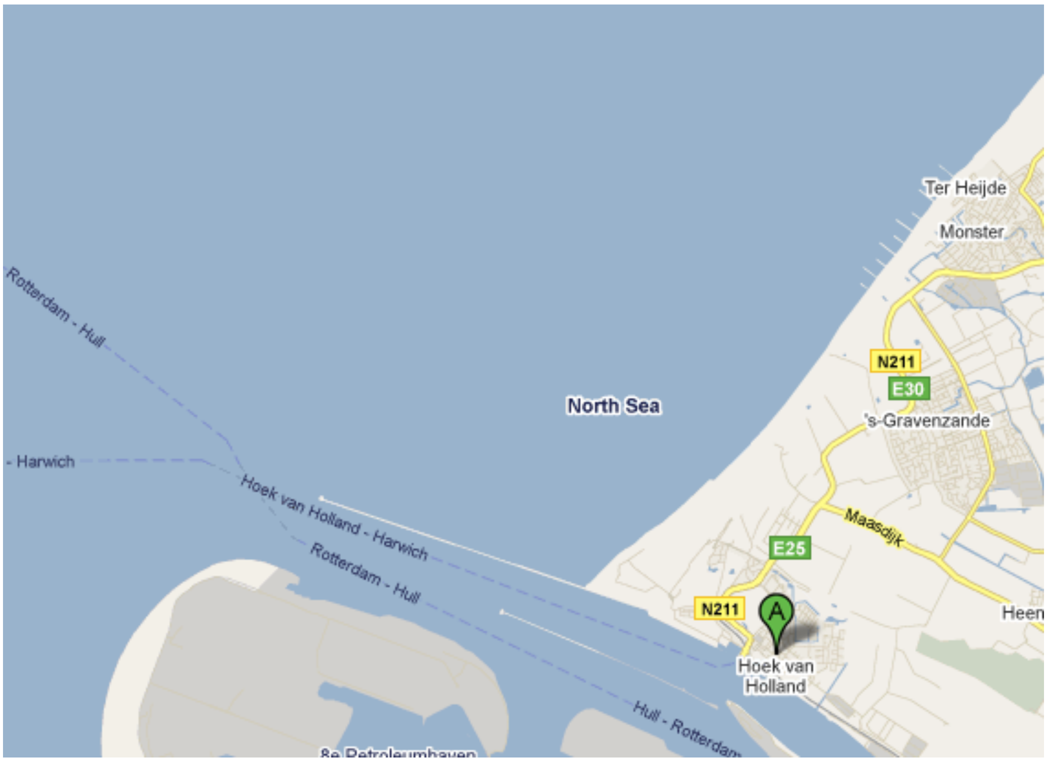


Figure 7-3: Hook of Holland and Ter Heyde: initial nourishment locations

The sand motor

- Design a Δ-form at Hook of Holland and a sawtooth at Ter Heyde
- 20 and 10 Mm3, that feed a yearly 1 to 2 Mm3 to the adjacent sections of the coast (wave-induced longshore transport);
- Contractors bid for a 30 year contract to nourish 1 Mm3/year split over both locations Hook of Holland and Ter Heyde.

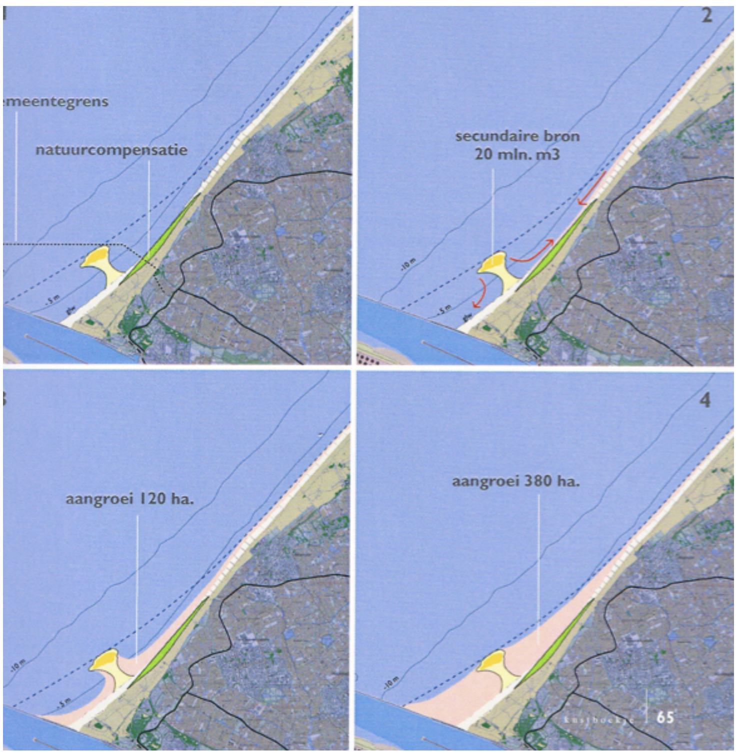


Figure 7-4: Morphological effects of Sand Motor project

The upper schematization illustrates an initial nourishment of 30 Mm^3 (20 and 10m3 on both locations) and a yearly maintenance of 1 Mm^3 over 30 years.

The lower schematization represents the traditional nourishment of 2 Mm^3 of two times a year.

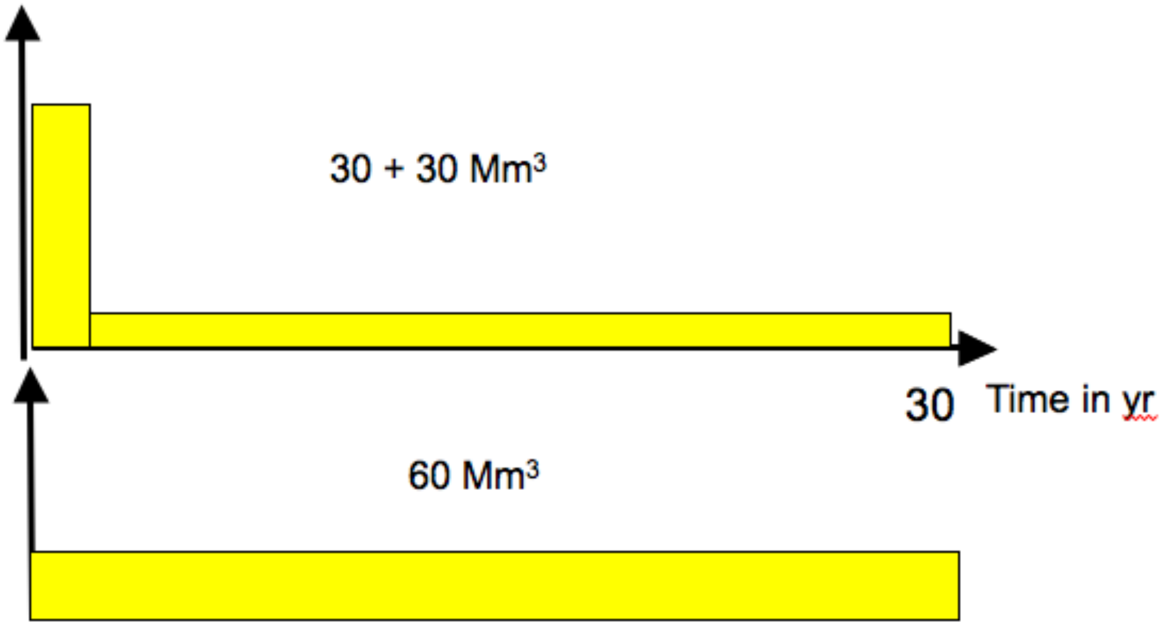


Figure 7-5: Quantities for the sand motor and traditional nourishment

Present value of a cash flow can be illustrated as follows.

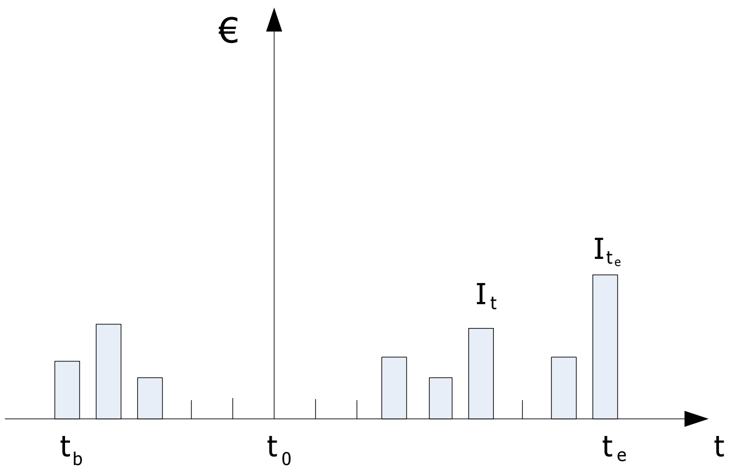


Figure 7-6: Cash Flow diagram, present value scheme

In formulas:

$$\text{Future: PV} = \sum_{t=t_0}^{t_e} I_t (1+r)^{t_0 - t}$$

$$\text{Past: PV} = \sum_{t=t_b}^{t_0} I_t (1+r)^{t_0 - t}$$

Present value (PV) of a constant cash flow can be represented as follows

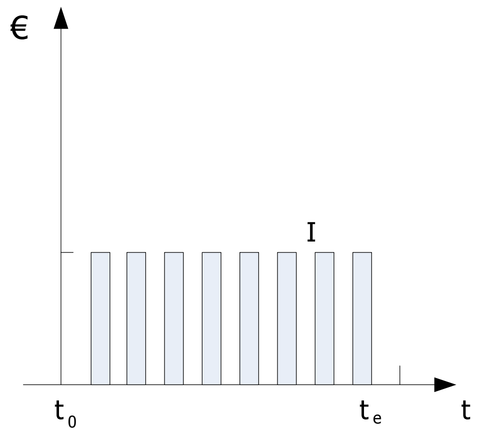


Figure 7-7: Constant cash flow diagram

$$PV = I * \sum_{t=t_0}^{t_e} (1+r)^{t_o - t}$$

$$PV = \frac{1}{r} * [1 - \frac{1}{(1+r)^{(t_e - t_0)}}]$$

By using the PV-factor the formula will be:

$$PV = PV\_factor(r, t_e) * I$$

PV_factor can be found in the Present Value tables (Brealey, et al., 2008)

The cost of the sand motor

- Suppose a price of 5 €/m3 and interest 5%
- Cost of two initial nourishment 20+10 Mm3=**150 M€** (op t=0)

- Cost of yearly maintenance 1Mm3 = 5 M€/yr
- Present value factor for 5% and 30 y = 15.37 (see table)
- Present value of yearly maintenance = 15.37 * 5 ≈ **77 M€**
- Present value Sand Motor 150 + 77 = 227 M€ in total

The cost of traditional nourishment

- No initial nourishment/yr
- Cost of double yearly maintenance 2Mm3 * 5 M€/yr = 10 M€/yr

- Present value factor for 5% and 30 y = 15.37
- Present value traditional nourishment = 10 x 15.37 = 154M€

Conclusion about costs of the Sand Motor project

- The sand motor is 1.5x more expensive (227 M€) than traditional nourishment (154M€), but allows "nature to nourish the beach"

7.3 Project Westerschelde Bridge (1986)

7.3.1 Introduction

The project is “Westerschelde Oeververbinding” shortly 'WOV' has a long history. Nowadays a tunnel is built as a cross-river-connection between Zeeuws Vlaanderen (Terneuzen) and Central Zealand (Ellewoutsdijk). The building-preparations started 1997.

Before that a bridge was planned. As an example of financial engineering we will analyze the consequences of several events that can happen, such as a longer construction period than was planned, the influence of inflation and the financial consequences of increased traffic by economic growth.

Some data:

Total investment:	I = ƒ 625 M
Discount rate:	r = 7%
Traffic:	Q = 5.15 M cars per year
Toll price:	P = ƒ 7 per car
Income:	Q * P = ƒ 36 M per year
Subsidy:	S= ƒ 35 M per year
Construction time:	5 years
Exploitation period:	25 years
Revenues (rev):	36 + 35 – 5 = ƒ 66 M per year, Comes from toll gates/ports

The cash flow diagram looks as follows:

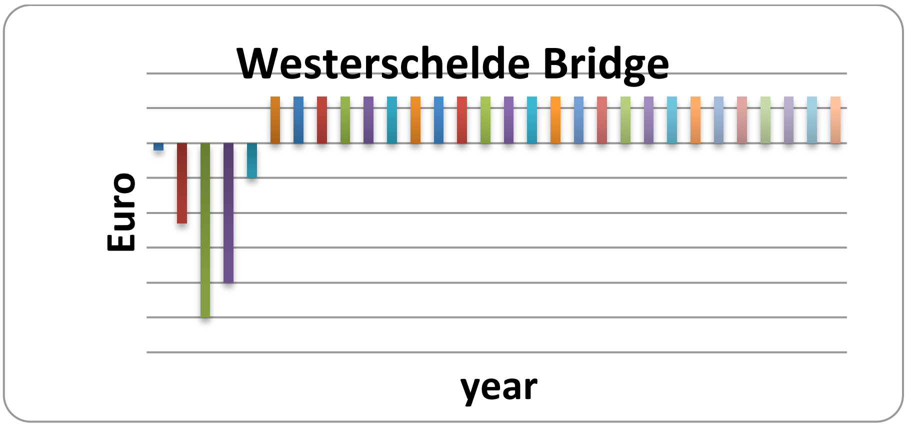


Figure 7-8: Cash flow diagram Westerchede Bridge

The construction period is about 5 year after which the exploitation will take place for 25 year. In the first 5 years you can see a net cash outflow. These sunk costs should be financed !
We can use the PV formula, respectively the constant cash flow formula.

In a spreadsheet

Toll company					
INVESTMENTS			in 1000's euros		
yr. Investment	**1**	**2**	**3**	**4**	**5**
Ground, site	-10.000	-15.000	-50.000	-50.000	
Concrete constr.		-100.000	-100.000	-50.000	
Steel constr.			-100.000	-100.000	-50.000
	-------	-------	-------	-------	-------
	-10.000	-115.000	-250.000	-200.000	-50.000
Cum. Credit demand	-10.000	-125.700	-384.499	-611.414	-704.213
Total investment	625000				
Discount rate, yr	7%				
PV Outflow	502.094				
Income					
Yr of inflow	**6**	**7**	**8**	**9**	**10**
Toll - € per vehicle	7				
Cars / day	14.286	14.286	14.286	14.286	14.286
Toll income	36.001	36.001	36.001	36.001	36.001
Subsidy	35.000	35.000	35.000	35.000	35.000
Maintenance	-5.000	-5.000	-5.000	-5.000	-5.000
	-------	-------	-------	-------	-------
Cash Inflow	66.001	66.001	66.001	66.001	66.001
Total Credit Demand	-687.507	-669.631	-650.504	-630.039	-608.141
FV 25 yr project t=5	769.136				
PV t=0	548.384				
NPV	46.290				
B/C ratio	1,09219312				

Table 7-1 Project evaluation Westerschelde Bridge

Because of the net cash outflow in the first years we see a typical infrastructure diagram in the next figure.

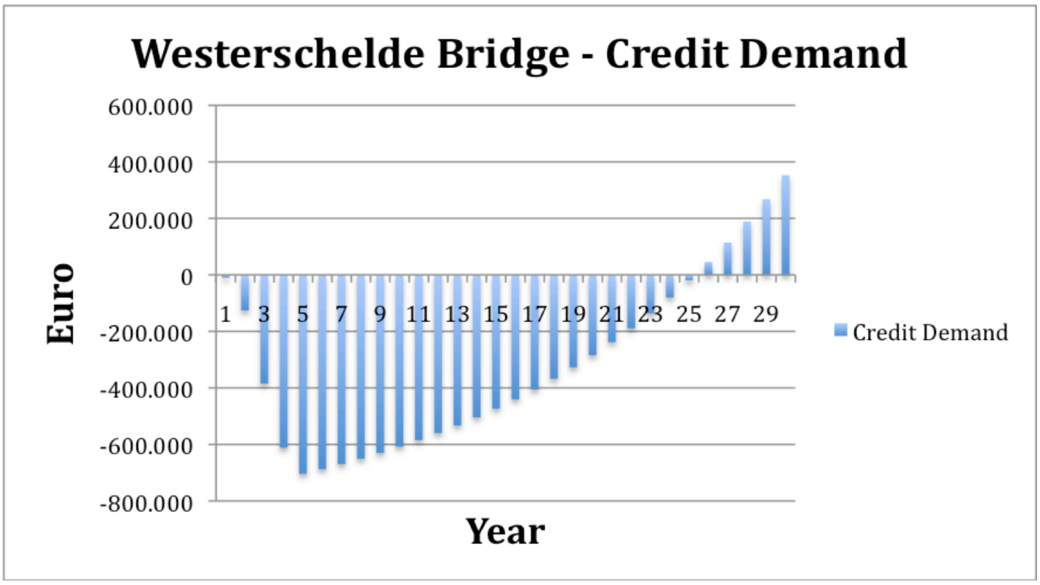


Figure 7-9: Credit demand Westerschelde Bridge

When the construction time of 5 years is followed by an exploitation period of 25 years the project appraisal will end up in the following results.
Results 5 year construction time

NPV_{5yr} = 548 – 502 = 46 kEuro

B/C_{5yr} = 548/502 = 1,0922

7.3.2 Westerschelde bridge - open earlier

From a financial point of view the year of opening the bridge makes a difference. The following schedule will help us to calculate the difference.

Figure 7-10: Cash flow schema – construction and exploitation

The constant cash inflow over 25 years is calculated at t=6 and after that the amount is discounted to year t=o, in which the discount rate is 7%.

When income is generated after 6 years the amount can be calculated by using formula:

$$CW(rev) = \frac{66}{r}\left(1 - \frac{1}{(1+r)^{25}}\right) * \frac{1}{(1+r)^{6}}$$

= 66 * annuity * PV_factor

= 942 * 0.815 * 0.666 (annuity and PV_factor see Appendix A in Brealey and Myers).

= 769 * **0.666**

= 512, if the bridge can be used after 6 years of construction.

If open earlier (after 5 years) we follow the same procedure, but the PV_factor will be different:

$$CW(rev) = \frac{66}{r}\left(1 - \frac{1}{(1+r)^{25}}\right) * \frac{1}{(1+r)^{5}}$$

= 769 * **0.713**

= 548 , if the bridge can be used after 5 years of construction.

The conclusion is that construction time is important for the financials of the project. When the bridge is open one year earlier the revenues will give a better return on investment (ca. 7%).

If we assume that the construction costs remain the same we find the results as shown below.

Results 5 year construction time	**Results 6 year construction time**
NPV_{5yr} = 548 – 502 = 46 kEuro	NPV_{5yr} = 512 – 502 = 10 kEuro
B/C_{5yr} = 548/502 = 1,0922	B/C_{5yr} = 512/502 = 1,0199

7.3.3 Sunk costs and sunk activities

Sunk costs are like spilled milk (Brealey, et al., 2008, p. 146). They are past and irreversible outflows. These cost are of importance for the liquidity of the project. So, sunk costs that are already paid.

Sunk activities: costs in the early stages of projects you have to pay before returns flowing in.

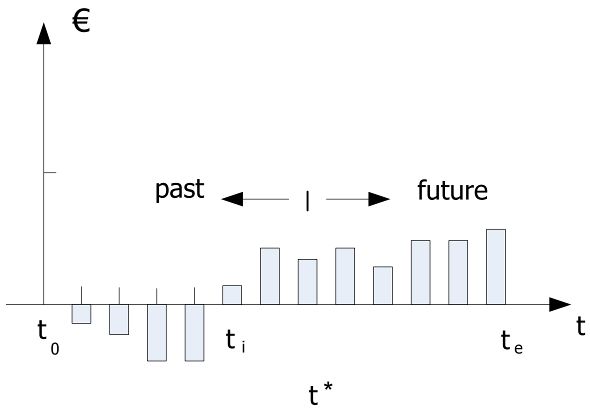


Figure 7-11: Sunk activities at time t=t*

Present Value of sunk activities at t^* can be calculated with the following expression:

$$NPV = \sum_{t=t_0}^{t_e} I_t (1+r)^{t_* - t}$$ for $t_* = t_e$ should: NPV>0

Figure 7-12: Cash flow schema - Sunk activities

The NPV on another moment, t* or t_e, is equal to the NPV on t_0 multiplied with a factor:

$$NPV_{t=1} = NPV_{t=0} * (1+r)^{t_1 - t_o}$$

so,

$$NPV_t = \sum_{t=t_0}^{t_e} I_t (1+r)^{t_*-t} = \sum_{t=t_0}^{t_e} I_t (1+r)^{t_0-t} * (1+r)^{t_*-t}$$

Construction costs (in 1000's euro's):

Toll company					
INVESTMENTS			in 1000's euros		
yr. Investment	**1**	**2**	**3**	**4**	**5**
Ground, site	-10.000	-15.000	-50.000	-50.000	
Concrete constr.		-100.000	-100.000	-50.000	
Steel constr.			-100.000	-100.000	-50.000
	-------	-------	-------	-------	-------
Construction	-10.000	-115.000	-250.000	-200.000	-50.000
Sunk costs	-10.000	-125.700	-384.499	-611.414	-704.213

Table 7-2: Sunk activities Westerschelde Bridge

Present Value = –502 M; Construction Investment = –625 M; Sunk Activities = –704 M

In which Sunk Activities: SA = PV * $(1+r)^n$ = 502 * $(1,07)^5$ = 704 (as we have seen before). These SA can be seen as the Future Value of all payments done.

Construction interest: Interest = 704 - 625 = ƒ 79 M !!! (that is, 79 / 625 = 12,6%)

The Profitability Index is NOT influenced by the choice of t_* as can be seen in the next derivation of formulas:

$$\frac{B}{C} = \frac{\sum_{t=t_i}^{t_e} I_t * (1+r)^{t_*-t}}{\sum_{t=t_0}^{t_i} I_t * (1+r)^{t_*-t}} = \frac{\sum_{t=t_i}^{t_e} I_t * (1+r)^{t_0-t} * (1+r)^{t_*-t_0}}{\sum_{t=t_0}^{t_i} I_t * (1+r)^{t_0-t} * (1+r)^{t_*-t_0}}$$

7.3.4 Westerschelde Bridge – Effect of traffic growth

We made some calculation with 7% discount rate). What will happen when traffic is increasing every year with 2%. The cash inflow will be higher, but how can we calculate that effect?

In the literature some formulas are derived for 'growing' perpetuities and annuities. When the discount rate is r and the cash flow is growing by g, than the estimate of present value can be calculated with factor 'r-g' (Brealey, et al., 2008: paragraph 3.3).

In the example of the Westerschelde Bridge, traffic is growing with 2% every year, the cash flow can discounted with a discount rate of : r'= 7-2= 5%.

$$PV_{revenues} = \frac{66}{r'} * \left(1 - \frac{1}{(1+r')^{25}}\right) * \frac{1}{(1+r')^6}$$

=1320*0.705*0.75
=930*0.75
=1320*0.705*0.75
=698 no growth during construction
NPV = 698-502= 196

$$\frac{B}{C} = \frac{698}{502} = 1,39$$

7.3.5 Westerschelde Bridge Resume – NCW and B/C

Situation 5 year construction
NCW= 548-502=46
B/C= 548/502=1.09

Situation 6 year construction
NCW= 512-502=10
B/C=512/502=1.02

Situation 2% growth of traffic
NCW= 698-502= 198
B/C= 698/502= 1.39

A one year delay gives a bad return on investment (-7%). On the other hand growth in traffic of 2% will give a far better financial result (30%).

7.3.6 Westerschelde Bridge – cost price evaluation

We can do the analysis with the cost price method. So, the total demand for capital (credit) will be €704 million (initial investment plus interest). When this amount of money is depreciated in 25 years of operation than the annuity will be expressed as flows

$$Annuity = \frac{r}{1-\left(\frac{1}{1+r}\right)^N} + Investment$$

in which: r = 7%
N = 25 year
Investment = € 704 million

The annuity factor can also be found on the internet.[1]

Annuity = 0,0858 * 704 = € 60,42 million/year

On the assumption of (14.286 *360) car rides per year the tariff will be

$$tariff_{Ann} = \frac{60,42 * 10^6}{14.286 * 360} = 11,75 \qquad \text{€/ride}$$

The maintenance costs are

$$tariff_{maintenance} = \frac{5 * 10^6}{14.286 * 360} = 0,97 \qquad \text{€/ride}$$

Thus, the total cost price of a car drive is

$$total\ cost\ price = tariff_{Ann} + tariff_{maintenance} = 12,72$$

[1] *see, http://www.principlesofaccounting.com/ART/fv.pv.tables/pvofordinaryannuity.htm*
For further information, see Wikipedia: http://en.wikipedia.org/wiki/Chunnel

The revenues per car ride are

Toll price	€ 7
Subsidy (35 * 10^6 / rides)	€ 6,81

Total revenue per ride	€ 13,84

The profit per car ride is

Profit = sales price – total cost price

= 13,81 – 12,72 = € 1,09

The profit margin can be calculated

$$1 + profit\ margin = \frac{sales\ price}{cost\ price}$$

$$= \frac{13,81}{12,72} = 1,086$$

Profit margin is 0,086

Should it be possible to compare these methods to each other?

$$\frac{B}{C} = \frac{sales\ price - tariff_{maintenance}}{tariff_{Ann}} = \frac{13,81 - 0,97}{11,75} = 1,0928$$

The result is almost the same as calculated earlier (1,09219312, see Table 7-1)

7.4 Linking the shores – the Chunnel project

7.4.1 Introduction

The **Channel Tunnel** (French: *Le tunnel sous la Manche*), known colloquially as the **Chunnel**, is a 50.5-kilometre (31.4 mi) undersea rail tunnel linking Folkestone, Kent near Dover in the United Kingdom with Coquelles, Pas-de-Calais near Calais in northern France beneath the English Channel at the Strait of Dover[2]. At its lowest point it is 75 m (246 ft) deep. At 37.9 km (23.5 mi), the Channel Tunnel has the longest undersea portion of any tunnel in the world, although the Seikan Tunnel in Japan is both longer overall, at 53.85 km (33.46 mi) and deeper, at 240 m (790 ft) below sea level.

Figure 7-13: Location Chunnel between UK and France

The tunnel carries high-speed Eurostar passenger trains, Eurotunnel Shuttle roll-on/roll-off vehicle transport—the largest in the world—and international rail freight trains. The tunnel connects end-to-end with the LGV Nord and High Speed 1 high-speed railway lines. In 1996 the American Society of Civil Engineers identified the tunnel as one of the Seven Wonders of the Modern World.

Ideas for a cross-Channel fixed link appeared as early as 1802, but British political and press pressure over compromised national security stalled attempts to construct a tunnel. However, the eventual successful project, organized by Eurotunnel, began construction in 1988 and opened in 1994. The project came in 80% over its predicted budget. Since its construction, the tunnel has faced several problems. Fires have disrupted operation of the tunnel. Illegal immigrants and asylum seekers have attempted to use the tunnel to enter Britain, causing a minor diplomatic disagreement over the location of the Sangatte refugee camp, which was eventually closed in 2002.

7.4.2 Arrangement

The British *Channel Tunnel Group* consisted of two banks and five construction companies, while their French counterparts, *France–Manche*, consisted of three banks and five construction companies. The role of the banks was to advise on financing and secure loan commitments. On 2 July 1985, the groups formed Channel Tunnel Group/France–Manche (CTG/F–M). Their submission to the British and French governments was drawn from the 1975 project, including 11 volumes and a substantial environmental impact statement.

Ten construction companies in the CTG/F-M GROUP did the design and construction. Five French construction companies in the joint venture group GIE Transmanche Construction undertook the French terminal and boring from Sangatte. Five British construction companies in the Trankslink Joint Venture undertook the English Terminal and boring from Shakespeare Cliff. TransManche Link (TML), a bi- national project organization, linked two partnerships. The Maître d'Oeuvre was a supervisory engineering body employed by Eurotunnel under the terms of the concession that monitored project activity and reported back to the governments and banks.

In France, with its long tradition of infrastructure investment, the project garnered widespread approval and in April the French National Assembly gave unanimous support and, in June 1987, after

For further information, see Wikipedia: http://en.wikipedia.org/wiki/Chunnel a public inquiry, the Senate gave unanimous support. In Britain, select committees examined the proposal, making history by holding hearings outside of Westminster, in Kent. In February 1987, the third reading of the Channel Tunnel Bill took place in the House of Commons, and was carried by 94 votes to 22. The Channel Tunnel Act gained Royal assent and passed into English law in July of that year. Parliamentary support for the project came partly from provincial members of Parliament on the basis of promises of regional Eurostar through train services that have never materialised; the promises were repeated in 1996 when the contract for construction of the Channel Tunnel Rail Link was awarded.

The Channel Tunnel is a build-own-operate-transfer (BOOT) project with a concession. TML would design and build the tunnel, but financing was through a separate legal entity: Eurotunnel. Eurotunnel absorbed CTG/F-M and signed a construction contract with TML; however, the British and French governments controlled final engineering and safety decisions. The British and French governments gave Eurotunnel a 55- (later 65-) year operating concession to repay loans and pay dividends. A Railway Usage Agreement was signed between Eurotunnel, British Rail and the Société Nationale des Chemins de Fer Français guaranteeing future revenue in exchange for the railways obtaining half of the tunnel's capacity.

Private funding for such a complex infrastructure project was of unprecedented scale. An initial equity of £45 million was raised by CTG/F-M, increased by £206 million private institutional placement; £770 million was raised in a public share offer that included press and television advertisements, a syndicated bank loan and letter of credit arranged £5 billion. Privately financed, the total investment costs at 1985 prices were £2600 million. At the 1994 completion actual costs were, in 1985 prices, £4650 million: an 80% cost overrun. The cost overrun was partly due to enhanced safety, security, and environmental demands. Financing costs were 140% higher than forecast.

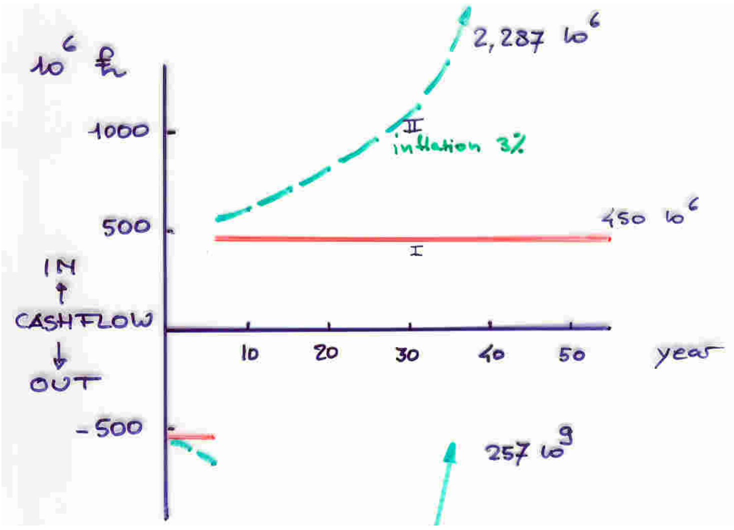


Figure 7-14: Project cash flow and effect of inflation

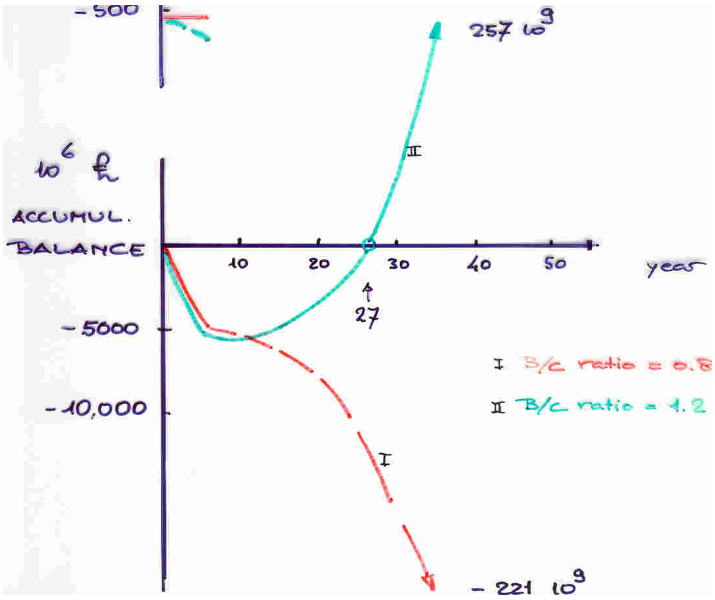


Figure 7-15: Accumulated cash flow by different B/C ratio

7.4.3 Economic performance

Shares in Eurotunnel were issued at £3.50 per share on 9 December 1987 (Wikipedia, 2009a). By mid-1989 the price had risen to £11.00. Delays and cost overruns led to the share price dropping; during demonstration runs in October 1994 the share price reached an all-time low value. Eurotunnel suspended payment on its debt in September 1995 to avoid bankruptcy. In December 1997 the British and French governments extended Eurotunnel's operating concession by 34 years to 2086. Financial restructuring of Eurotunnel occurred in mid-1998, reducing debt and financial charges. Despite the restructuring *The Economist* reported in 1998 that to break even Eurotunnel would have to increase fares, traffic and market share for sustainability. A cost benefit analysis of the Channel Tunnel indicated that there were few impacts on the wider economy and few developments associated with the project, and that the British economy would have been better off if the tunnel had not been constructed.

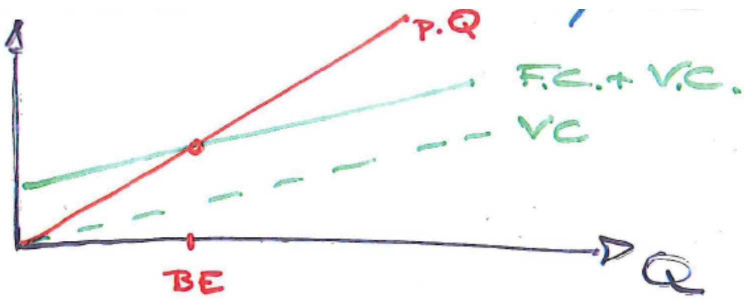


Figure 7-16: Example of break-even analysis

Under the terms of the Concession, Eurotunnel was obliged to investigate a cross-Channel road tunnel. In December 1999 road and rail tunnel proposals were presented to the British and French governments, but it was stressed that there was not enough demand for a second tunnel. A three-way treaty between the United Kingdom, France and Belgium governs border controls; with the establishment of *control zones* wherein the officers of the other nation may exercise limited customs and law enforcement powers. For most purposes these are at either end of the tunnel, with the French border controls on the UK side of the tunnel and vice versa. For certain city-to-city trains, the train itself represents a control zone. A bi-national emergency plan coordinates UK and French emergency activities.

In 1999 Eurostar posted its first ever net-profits, having previously made a loss of £925m in 1995.

8 Markets of consumer goods

The characteristics of markets can be seen on elasticities with respect to price or income. **Price elasticity of demand** (**PED** or $\mathbf{E_d}$) is a measure used in economics to show the responsiveness, or elasticity, of the quantity demanded of a good or service to a change in its price. More precisely, it gives the percentage change in quantity demanded in response to a one percent change in price (holding constant all the other determinants of demand, such as income). It was devised by Alfred Marshall.

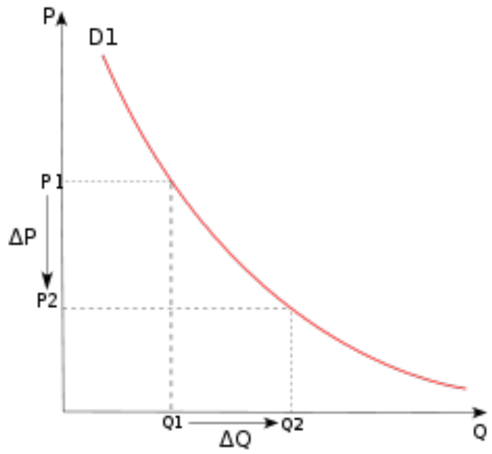


Figure 8-1 Price elasticity of demand
PED is derived from the percentage change in quantity (%ΔQd) and percentage change in price (%ΔP).

Price elasticities are almost always negative, although analysts tend to ignore the sign even though this can lead to ambiguity. Only goods which do not conform to the law of demand, such as Veblen and Giffen goods, have a positive PED. In general, the demand for a good is said to be inelastic (or relatively inelastic) when the PED is less than one (in absolute value): that is, changes in price have a relatively small effect on the quantity of the good demanded. The demand for a good is said to be elastic (or relatively elastic) when its PED is greater than one (in absolute value): that is, changes in price have a relatively large effect on the quantity of a good demanded.
Revenue is maximised when price is set so that the PED is exactly one. The PED of a good can also be used to predict the incidence (or "burden") of a tax on that good. Various research methods are used to determine price elasticity, including test markets, analysis of historical sales data and conjoint analysis.

Product	$\boldsymbol{\varepsilon_p}$	
Beer	0.30	inelastic
Steak	1.90	elastic
Tobacco	0.47	
Restaurant	2.27	elastic
Shoes	0.73	
Jewels	0.41	
Electricity	0.13	inelastic
Water	0.20	inelastic
Rent (housing)	0.18	inelastic
Kitchen (apartment)	0.67	
Telephone	0.26	inelastic
Legal aid	0.37	

Figure 8-2: Overview of some price elasticity's of demand

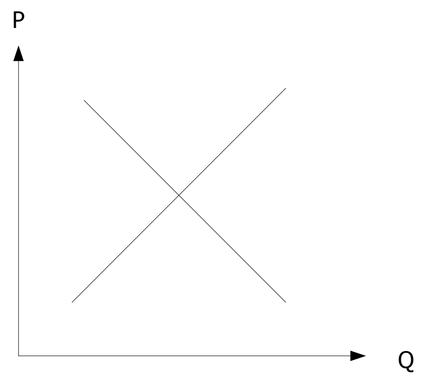
Example:
As a result of an increase in the price of consumer-tax on a bottle of €14,- there will be a decrease in distribution of $42*10^6$ to $31*10^6$ liter

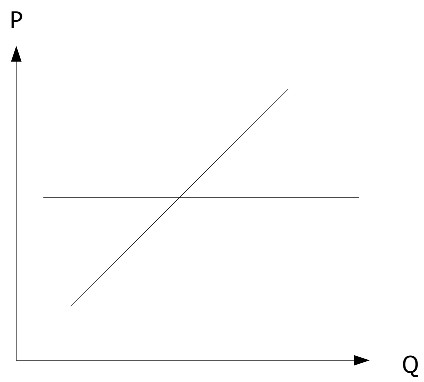
Price elasticity: $\varepsilon_p = -\frac{\Delta Q}{\Delta P} * \frac{P}{Q} = -\frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}} = -\frac{\frac{-11}{42}}{\frac{1}{14}} = -\frac{-26\%}{7\%}$

$$\varepsilon_p = -\frac{26\%}{7\%} = 3.67$$

Product	ε_p	
Alcoholic liquors	0.29	inelastic
Restaurant	1.61	elastic
Movie/cinema	0.81	
Books	1.67	elastic
Dentist	0.38	
Furniture	2.60	elastic
Funeral	0.48	
Car repair	1.03	
Taxi	1.14	
Toys	0.59	

Figure 8-3: Income elasticity





Supply-demand curve total market → supply-demand curve one supplier

Demand and supply

A supplier wants to supply a quantity Qs against a price P_s
$P_s = a + b*q_s$
A demander wants to buy q_d goods at a certain price per good p_d
$p_d = c - d*q_d$

Both parties agree when $p_s = p_d = p'$ and $q_s = q_d = q'$

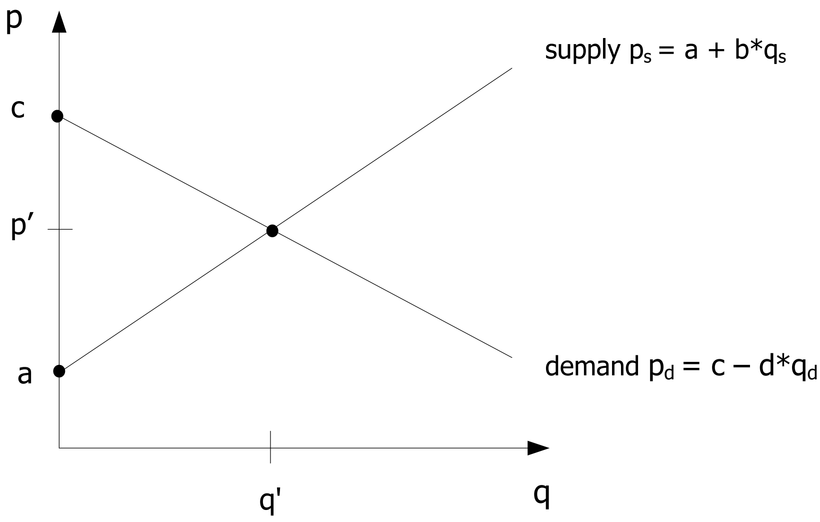


Figure 8-4: Representation of supply and demand

After equating the prices and the quantities, it results in:

$$q' = \frac{c-a}{b+d}$$

$$p' = \frac{bc+ad}{b+d}$$

One of the problems within economics is that the supply- and demand curve cannot be determined. Only the movement of the point of equilibrium (p',q') can be observed. Because of a general price increase of supply, both p' and q' react according to the demand curve.

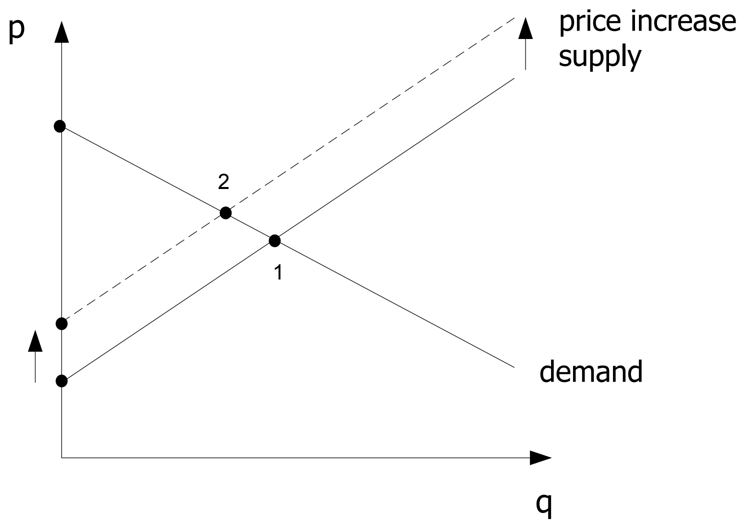


Figure 8-5: Demand change on price rise

In most cases, there are a great number of suppliers (M) and demanders (N) active in the market. Demand and supply are in that market equal to

$$Q_d = Nq_d \qquad\qquad Q_s = Mq_s$$

The supply curve of the M suppliers together becomes

$$p_s = a + b * \frac{Q_s}{M}$$

and the demand curve becomes

$$p_d = c - d\,\frac{Q_d}{N}$$

The market is at equilibrium when $p_s = p_d = p'$ and $Q_s = Q_d = Q'$

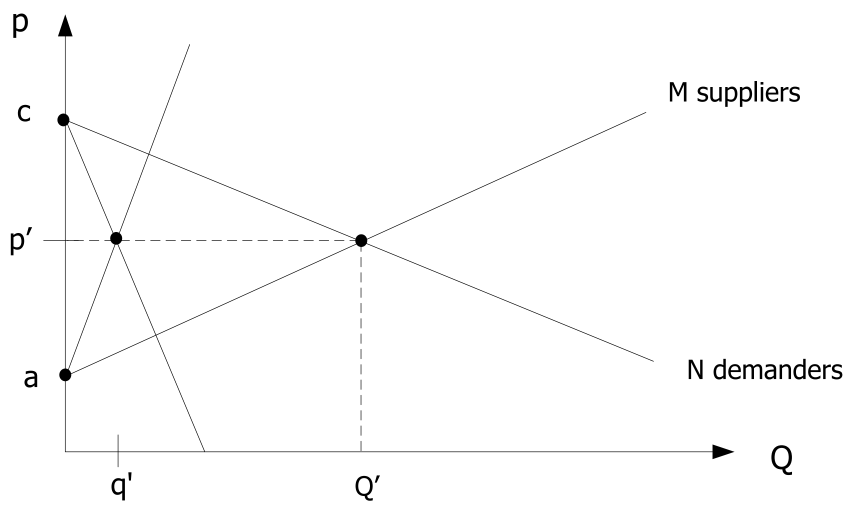


Figure 8-6: Aggregated demand and supply of goods

This results in the following:

$$Q' = \frac{c-a}{\frac{b}{M}+\frac{d}{N}} \quad p' = \frac{c\frac{b}{M}+a\frac{d}{M}}{\frac{b}{M}+\frac{d}{M}}$$

For only one supplier the market looks totally different from the one in which he only had one demander.
The demand curve is unchanged

$$p_s = a + bq_s$$

The total demand is almost saturated by the M-1 other suppliers

$$p_d = c - \frac{d}{M}\left\{\frac{M-1}{M}Q' + q_s\right\}$$

$$= c - \frac{d}{M}\frac{M-1}{M}Q' - \frac{d}{M}q_s$$

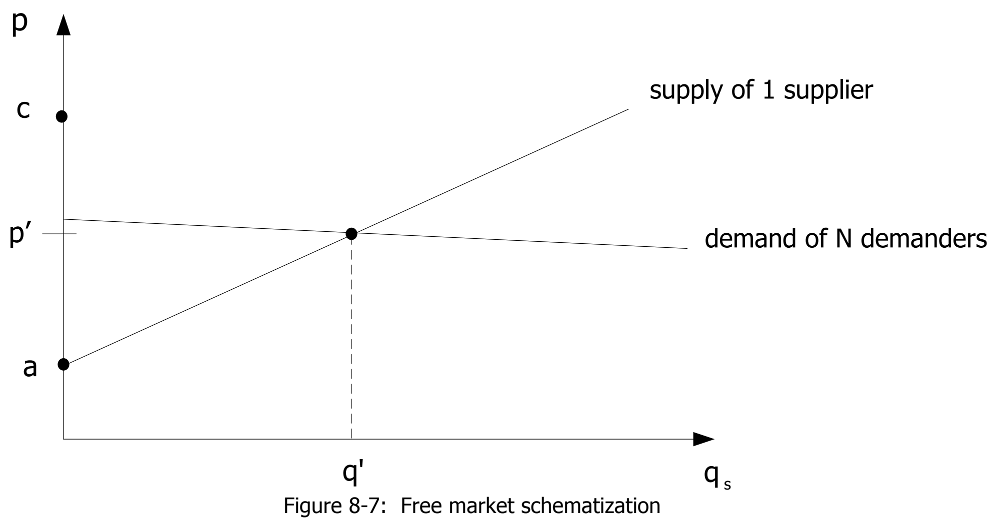


Figure 8-7: Free market schematization

One supplier has almost no power to influence the market price with his own supply. The slope of the demand curve is $\frac{d}{M}$. M → ∞ results in $\frac{d}{M}$ → 0, so the influence is almost zero.

This type of market can be characterized as free market.

One single demander has also almost no influence on the market price. The slope of the demand curve is $\frac{b}{N}$ and the value approaches the zero as the number of demander's increases.

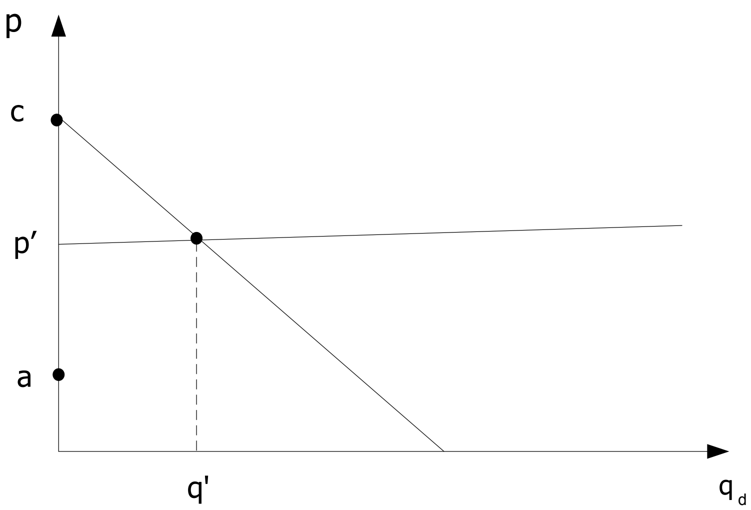


Figure 8-8: Increasing demand

9 Capital markets

9.1 Introduction

A firm's basic resource is the stream of cash flows produced by its assets. When the firm is financed entirely by common stock, all those cash flows belong to the shareholders. When it issues both debt and equity securities, it splits the cash flow in two streams, a relatively safe stream that goes to the debt holders and a riskier stream that goes to the stockholders.

The firm's mix of debt and equity financing is called its capital structure.

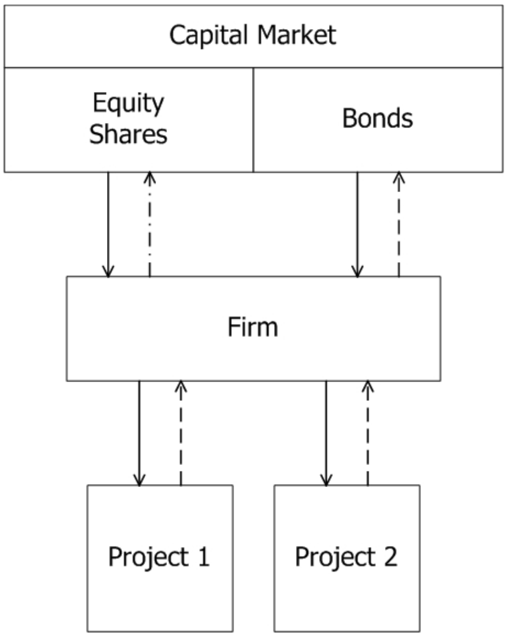


Figure 9-1: Classical way of financing projects

DEBT
- Bonds (Firm sells financial "paper" and promises to pay interest and 'par value' at the end of the term)
- Bank loan
- Fixed payment interest
- Repaid at maturity, when the loan agreement ends

EQUITY
- Shares (Firm doesn't pay back. Investor can sell the shares to another person)
- Share of the profit is uncertain
- No repayment
- No maturity

Uncertainty and return on investment always come together. In finance, ***seniority*** refers to the order of repayment in the event of bankruptcy. Senior debt must be repaid before subordinated debt is repaid. Bonds that have the same seniority in a company's capital structure are described as being ***pari passu***. Based on the level of certainty, that is called Seniority, the following ordered list of securities comes to front (the more certain securities are showed on top):

1. Secured debt (e.g. mortgages)[3]
2. Debt
3. Subordinated debt (better ROI, higher risk)
4. Convertible debt
5. Preferred stock / shares
6. Stock / shares

9.2 Market value of debt

The value of an eternal cash flow depends on the rate of interest.

$$\frac{Y_0}{r} = B, that\ is\ called\ market\ value$$

Suppose Y_B = 50 euro and r=5%, than

$$\frac{Y_B}{r} = \frac{50}{0{,}05} = 1000$$

If the interest falls to r=2,5%

$$\frac{Y_B}{r} = \frac{50}{0{,}025} = 2000$$

The other way around, when the interest rises to r=10%

$$\frac{Y_B}{r} = \frac{50}{0{,}10} = 500$$

When debt is issued the book value D is equal to the market value B, that is (at t0)

$$Y_B = r_{t_0} * D$$
$$B_0 = \frac{Y_B}{r_{t_0}} = D$$

Some later at tn

$$B_n = \frac{Y_B}{r_{t_n}} \neq D$$

It is more complicated in reality:

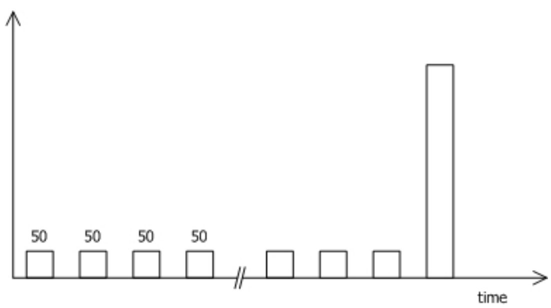


Figure 9-2: Cash flow and repayment nominal value

In the figure a cash flow series is shown (50's) and an amount for repayment (1000).
The market value can be shown by the next expression

[3] *In the USA you can definitely leave your house without further obligations. You only have to return the keys to the bank.*

$$B_n = \frac{Y_B}{r} * \left(1 - \frac{1}{(1+r)^n}\right) + \frac{nominal\ value}{(1+r)^n}$$

In which: r = real rate + inflation + risk (see also paragraph about inflation)

On the share market:

$$\frac{Y_{share}}{i} = S, which\ is\ the\ market\ value\ of\ stock$$

For small value of interest: i = real rate +inflation + risk

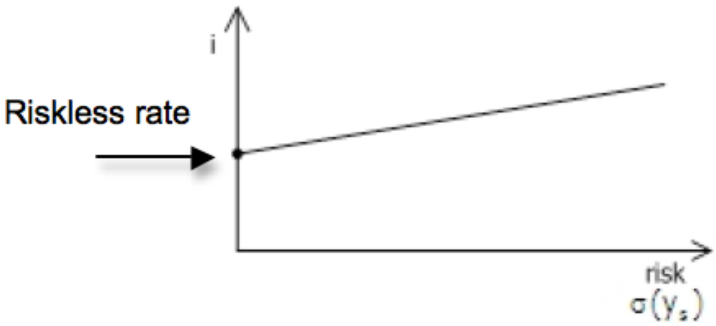


Figure 9-3: Security market line

The security market line shows the relation between risk and return (Brealey, et al., 2008: chapter 9). The riskless rate is often the return on Treasury Bills.

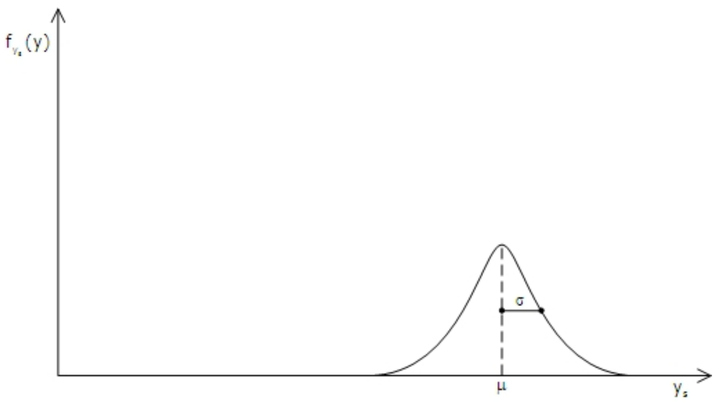


Figure 9-4: Probability density function of part of the income on shares

The probability density function (PDF) is the derivative of the probability distribution function. In this PDF the μ is the average and σ is the spread (degree of risk) (Vrijling, 2006: p2-6).

9.3 Classical point of view of the owner - leverage

Leverage is an financial mechanism for maximizing profits (hot shower). But there are some risks (the cold shower). How does it work ?

Project can be financed by equity (E) and debt (D). The ratio between those is called leverage, λ.

$$\lambda = \frac{D}{D+E}$$

If the project rate of return R>r, then the income for the owner is the total income of the project minus debt interest, that is

$$R * (E + D) - (r * D)$$

then the rate of return for the owner will be "income owner / equity":

$$\frac{R * (E + D) - (r * D)}{E}, that\ is\ R + (R - r) * \frac{\lambda}{1-\lambda}$$

In an example: R=15% and r=5% (conditions for the hot shower)

$$When\ the\ leverage\ \lambda = 0,5:$$

$$the\ return\ on\ investment\ of\ the\ owner\ is:\ 0,15 + (0,15 - 0,05) * \frac{0,5}{1-0,5} = 0,25$$

$$When\ the\ leverage\ \lambda = 0,9:$$

$$the\ return\ increases\ considerably: 0,15 + (0,15 - 0,05) * \frac{0,9}{1-0,9} = 1,05$$

By increasing the leverage from 0,5 to 0,9 the return for the owner has increased from 25% to 105%. This is called the hot shower.

But when the project is not doing very well and the overall return, R, of the project diminishes to 2%, then:

$$When\ the\ leverage\ \lambda = 0,9 > \quad 0,12 + (0,02 - 0,05) * \frac{0,9}{1-0,9} = -0,15$$

the cold shower is dripping…

Before the credit crunch, some project were started where R was 5,5% and r was 5%. To increase the return extremely high leverages applied. So, small events could start the cold shower.

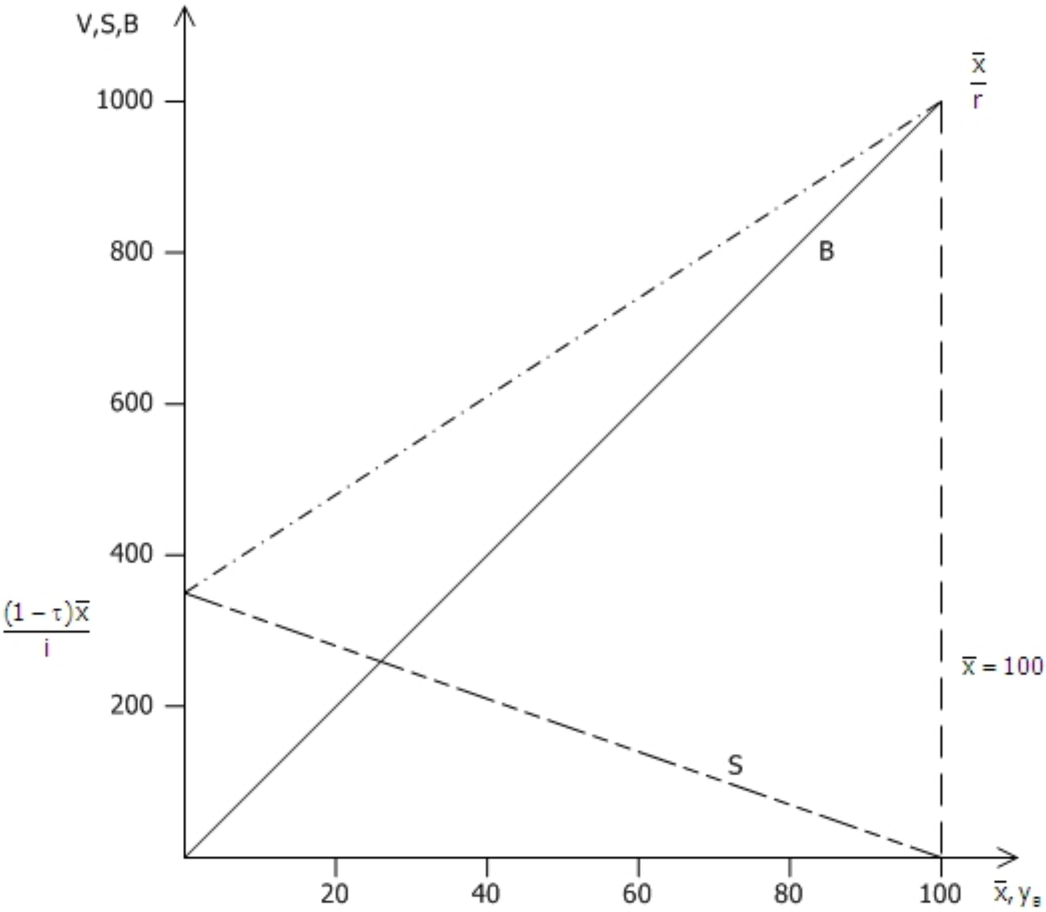
$$Credit\ crunch\ \lambda = 0,99\ !! \quad 0,055 + (0,055 - 0,05) * \frac{0,99}{1-0,99} = 0,55$$

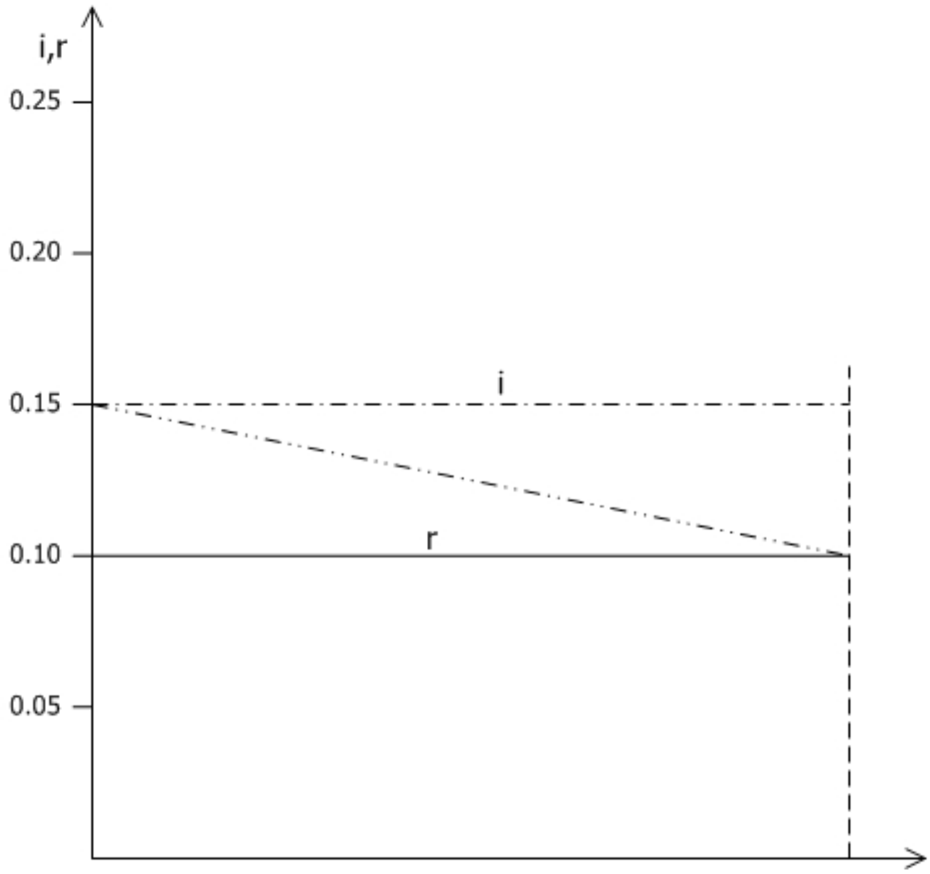
The balance looks like:

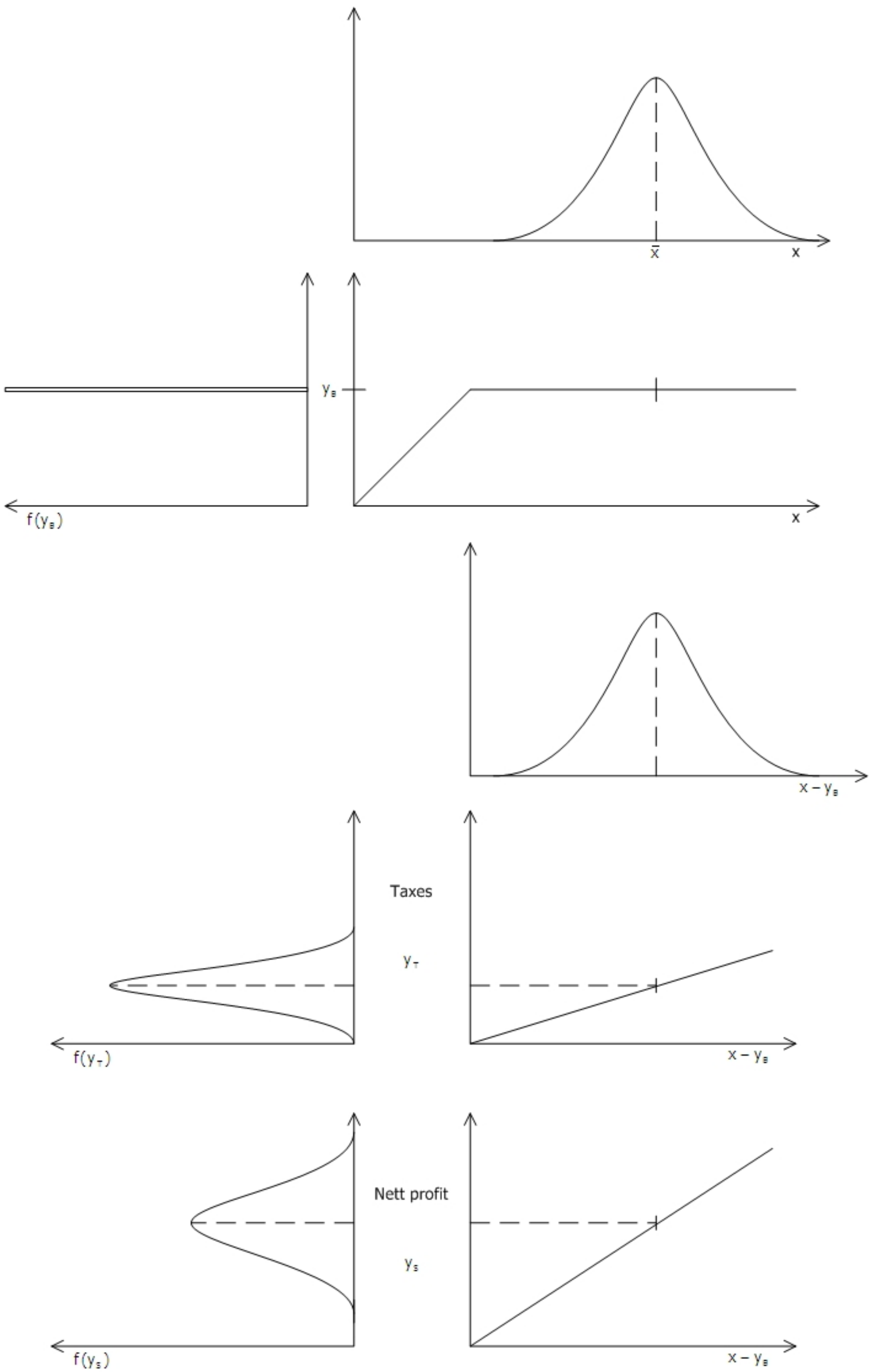
Balance			
Assets	1000	E	10
		D	990

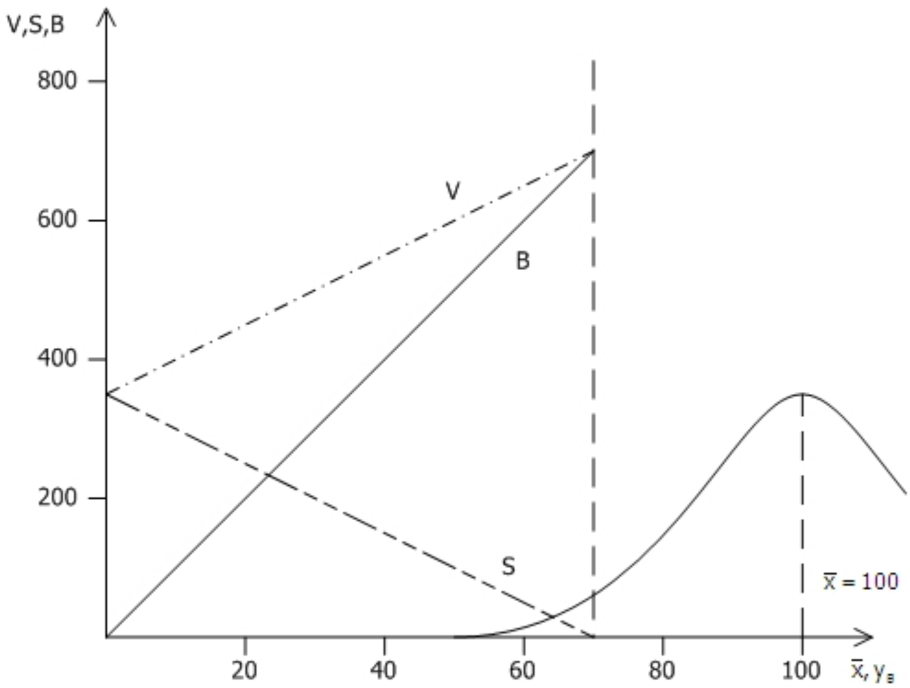
Figure 9-5: Model of balance sheet

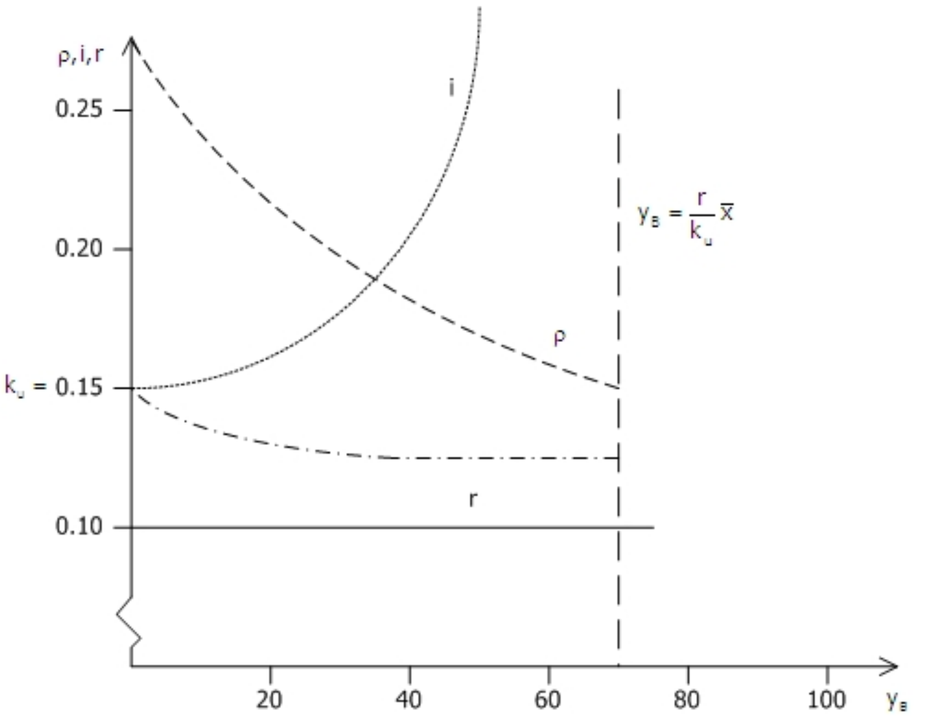
On the balance sheet you can see 1% Equity, which means risky business. If R drops to 5% the profit is wiped out.

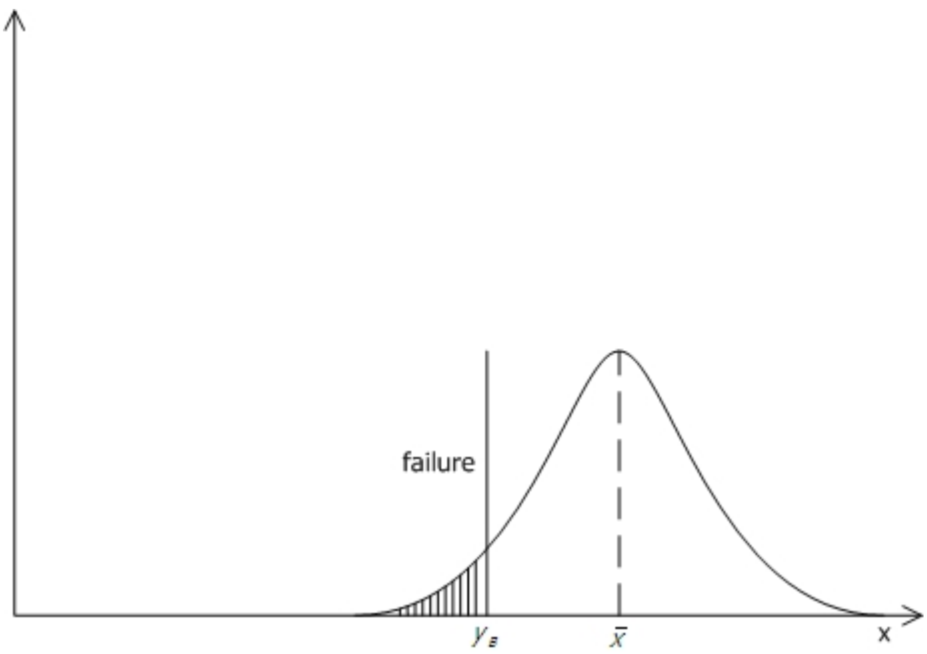

V,S,B
1000
800
600
400
200
$\frac{(1-\tau)\bar{x}}{i}$
$\frac{\bar{x}}{r}$
B
S
$\bar{x} = 100$
20
40
60
80
100
$\bar{x}, y_B$

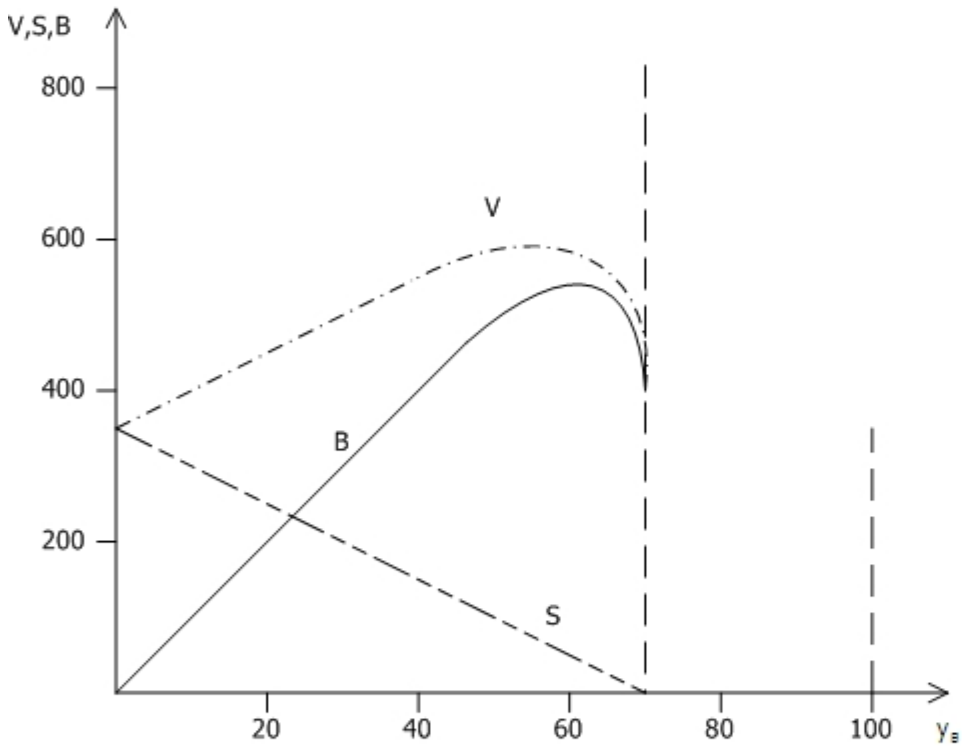

i,r
0.25
0.20
0.15
0.10
0.05
i
r

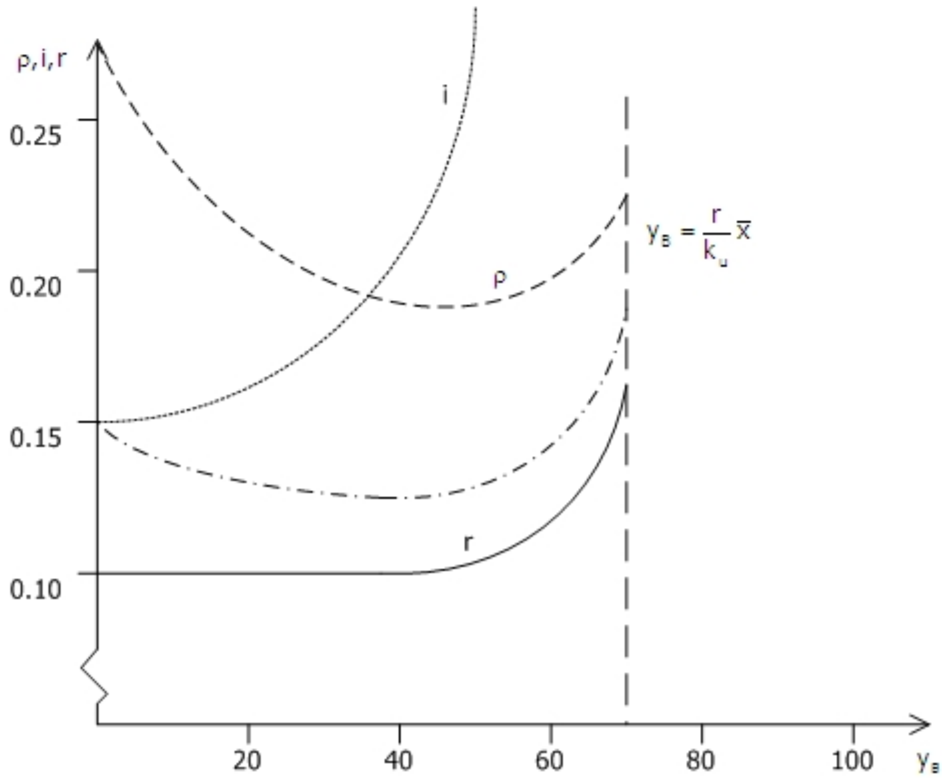

x̄
x
y_B
f(y_B)
x
x − y_B
Taxes
y_T
f(y_T)
x − y_B
Nett profit
y_s
f(y_s)
x − y_B


V,S,B
800
600
400
200
V
B
S
$\overline{x} = 100$
20
40
60
80
100
$\overline{x}, y_B$


ρ,i,r
0.25
0.20
k_u = 0.15
0.10
i
ρ
r
$y_B = \frac{r}{k_u}\overline{x}$
20
40
60
80
100
y_B


failure
y_B
$\bar{x}$
x


V,S,B
800
600
400
200
V
B
S
20
40
60
80
100
y_B


ρ,i,r
0.25
0.20
0.15
0.10
i
ρ
r
$y_B = \frac{r}{k_u}\bar{x}$
20
40
60
80
100
y_B

10 Economic aspects of the Life Cycle Approach

Author: prof. ir drs J.K.Vrijling, Delft University of Technology, Delft

10.1 Introduction

Only a few decades ago Public or Municipal Port Authorities designed and built their ports and the related infrastructure with quality as a first priority. Quality was interpreted to mean a technically impeccable design leading to structures with a guaranteed lifetime of 50 year or more. The scientific theory of maintenance was developed to optimize structures taking into account investment, management, maintenance and risk.
The main problem with this sophisticated approach of costs is that it looks to the technical lifetime only. However economists know that beside the costs also the revenue requires attention. In a world that tends to change from public to private investment revenue even gets most attention. The economic lifetime, that is generally shorter than the technical, becomes then the guiding principle of the life cycle approach.
Defining the client and looking to the entire process of sea transport, (un)loading and inland transport from his point of view and his time frame of change will be of decisive importance.

10.2 Theory of Maintenance

In the field of mechanical engineering considerable progress has been made with the scientific approach of the maintenance of machinery. Maintenance is defined as consisting of two activities: inspection and repair. Inspection implies the observation of the actual state of the machine or structure. Repair means the restoration of the artifact to its original state.
Two main classes of maintenance are discerned: corrective and preventive maintenance. When a corrective maintenance strategy is applied, the artifact will be repaired after failure. Inspection is omitted. In a preventive maintenance scheme the structure is preferably repaired before failure occurs. To this end running time or operational hours are observed and the structure is repaired at fixed time intervals or after a certain number of operational hours. The likelihood of failure in the intervals between repairs should be acceptably small.
In a more refined strategy the state of the structure or machine is inspected at such intervals. On the basis of the inspection result the decision to repair is taken.
The optimal cycle of inspection and repair is determined by minimization of the present value of the sum of the failure costs, inspection costs and repair costs. In these optimization models simple probability distributions for the time to failure such as the exponential distribution are frequently used. Sometimes this constant failure rate model is refined into a "bathtub" curve that gives the failure rate as a function of time.
The driving force behind the decision to opt for preventive maintenance is the consequence of failure. If the consequence of failure is large, one would prefer regular inspection in order to plan repair and to avoid catastrophic failure.

In the mechanical engineering problems described in the literature the failure rate is seldom related to the environmental conditions or loads. In the case of port structures damage and the maintenance consequently required is often initiated by extreme storm conditions. It is therefore imperative to relate the maintenance of port structures to environmental and other loadings.

10.3 Maintenance in Port Engineering

Firstly it seems that responsible port authorities will not rely on corrective maintenance in the case of port infrastructure, because serious damage or failure of the structures will most certainly cause (partial) closure of the port. This means not only lost revenue but also a damaged reputation. Therefore preventive maintenance is advised where repair is initiated as soon as the inspection indicates that the probability of failure has become too high.

Secondly the usual approach of maintenance using a direct assessment of the failure rate, based on the observation of the failure of a great number of artifacts that are kept running over time, may be practical for mechanical or electrical equipment but is impossible for hydraulic engineering structures like breakwaters, bottom protections or quay walls. These structures are unique, insufficient in number and too rarely failing to provide a homogeneous statistical base.
Here it is proposed to base the analysis on the engineering design approach that applies the analysis of the ultimate and serviceability limit states (ULS and SLS respectively). One class of ULS and two classes of SLS can be discerned:

ULS	models the failure of the structure under extreme loads S
SLS	describes the deterioration of the resistance R of the structure under the ever-present loads (e.g. fatigue or settlement)
SLS	analyses the functional performance of the structure under the ever-present loads (e.g. wave transmission by a breakwater)

The probability of failure is given by the probability of exceeding the ULS defined by $Z=R-S<0$. Reliability theory will provide the failure probability if the probability density functions (PDF's) of load S and resistance R are known. Here already an interesting difference in the failure probability as a function of time becomes evident. If the loads pose the major uncertainty (e.g. the loading by extreme storm waves) the failure probability will be constant over time as every year the 1/100 year load may hit the structure. If however the major uncertainty stems from the resistance (e.g. an earth structure with fixed loading) then the failure probability will be a rapidly falling function of time, because the survival of the first loading already proves that the structure is fit and every subsequent safely resisted loading reinforces the idea of a reliable structure.
The main concern of maintenance is that the resistance of most structures deteriorates over time by settlement, fatigue, corrosion, erosion, etc. The decreasing resistance as described by the first class of SLS's will force the failure probability up all other factors remaining the same. The common approach of these limit states with the additional help of reliability theory will overcome the problem that a "bathtub curve" of the average breakwater is not available. Moreover an average failure rate would not be of much value in a specific case.
The proposed probabilistic approach will be illustrated here with a simple case of channel siltation. It is assumed that every year the channel silts up with a layer z_i.
The value of z_i is normally distributed with a positive mean and standard deviation, defining the siltation process as a random walk phenomenon. The actual depth of the channel at t is now given by the SLS equation: $d(t)= d(0)-\sum z_i$. The channel will become shallower over time by random walk. The siltation makes the grounding of a ship in the channel ever likelier. The probability of grounding as a function of time can be calculated using the ULS equation: $Z(t)= d(t)-d_{ship}$ The two random variables $d(t)$ and d_{ship} define the probability $Pr(Z(t)<0)$ that $Z(t)<0$ and grounding occurs, as a function of time. The port authorities will however not wait until the probability of grounding becomes too large and implement a regular dredging operation to restore the channel to d(0). This is an example of preventive maintenance. If the dredging operation requires a considerable amount of money, a regular depth sounding operation will be put in place to optimize the start of the dredging. And if depth sounding is expensive or difficult to perform, a simple monitoring of the depth at some key sections will be chosen to signal the necessity of a full echo sounding. This shows that depending on the relative costs a sequence of monitoring some proxy variables to decide on a full inspection of the state of the structure to support the decision on repair might be efficient.

MONITORING →	INSPECTION →	REPAIR
Time Use Load State	Resistance or state	

Table 10-1.The sequence of monitoring, inspection and repair

The subsequent choices to be made in maintenance are modeled in next figure.

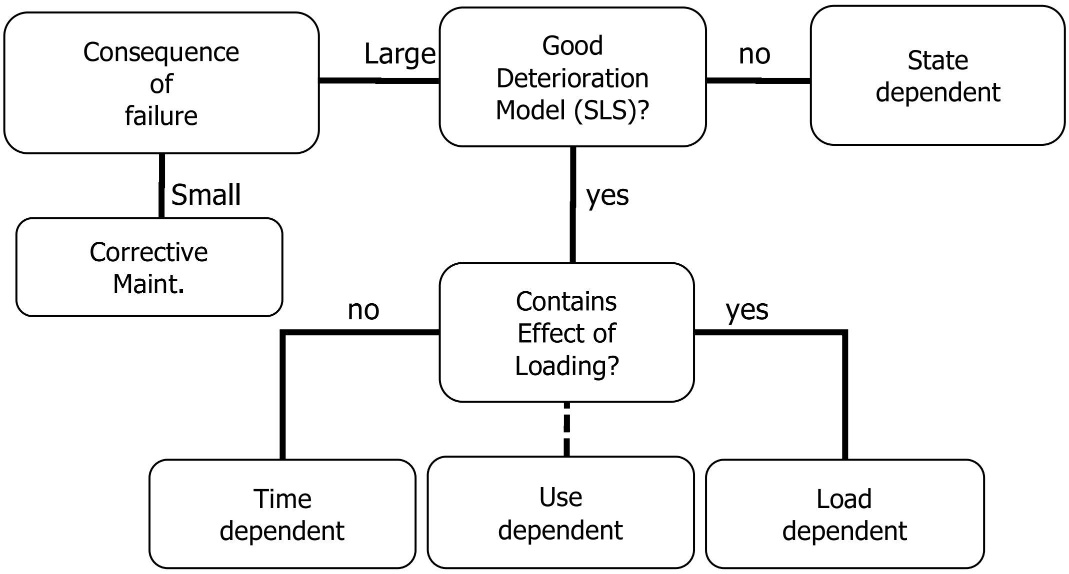


Figure 10-1:The choice between maintenance strategies

Depending on the consequences of failure one opts for preventive maintenance. The next question is the availability of a good SLS model that predicts the reduction of the resistance. If it is not available monitoring and inspection of the state is necessary. If the resistance can be predicted on the basis of time, use or load these proxies can be monitored to signal the need of a thorough inspection and probably repair.

The value of inspection is that it sharpens the knowledge of the actual state of the structure after a period of educated guessing based on the SLS-model extrapolation and supported by the monitoring of the proxy variables.
The optimal sequence and timing of monitoring, inspection and repair results from the minimization of the present value of the sum of monitoring cost, inspection cost, repair cost and risk (probability x consequence).

10.4 Life Cycle Cost

Recently much attention has been given to the advantage of life cycle cost analysis. The main motive may have been avoiding a reduction of investment that is later causing high repair costs or extreme removal expenses at the end of the productive life. However some argue that the damage to the environment of such 'unsustainable' investment decisions may be even higher than the economic costs suggest.
If we limit ourselves to the costs and leave the more subjective environmental judgments aside, it will be argued that the life cycle approach does not provide major new insights. The main reason is the relatively small contribution of the maintenance costs to the life cycle cost. If a technical life time of 50 year is assumed for this class of fixed structures, the cost of depreciation and interest exceeds the maintenance expenses both taken as a fraction of the investment. From the experience of the Dutch Rijkswaterstaat the following order of magnitude of yearly maintenance cost is estimated for four classes of structures

Type of structure	Maintenance cost/ investment
Reinforced concrete	0.5 – 1.0 %
Steel	1.0 – 2.0 %
Mechanical	2.0 – 3.0 %
Revetments, protections	0.5 – 1.0 %
Average Rijkswaterstaat	~ 0.5 %

Table 10-2: Maintenance cost for civil engineering structures

If an interest rate of 4% is assumed and the removal cost after 50 year is taken equal to the investment, the present value of the removal cost is ~11% of the investment. The risk is estimated by setting the consequence of failure at 100 times the investment and the probability of failure at 10^{-4} per year. The following table gives an overview of the life cycle costs of a fixed port structure in absolute as well as relative terms

Cost type	Cost per year / investment	Relative to total
Depreciation in 50 y.	2.0 %	24 %
Interest 4%	4.0 %	49 %
Repair & maintenance	1.0 %	12 %
Risk	1.0 %	12 %
Removal cost	0,22%	3 %
Total	**8.22%**	**100 %**

Table 10-3: Fixed costs for civil engineering structures

The table shows that maintenance and removal together constitute only 15 % of the total fixed cost. Generally speaking fixed asset costs form only 15 – 40 % of the total yearly costs of any organization, because wages and salaries account for over 50%. It may therefore be argued that the omission of maintenance and removal cost from design considerations concerns maximally only 7-8 % of the total cost. The theory of maintenance treated in the previous paragraphs aims to optimize these costs by choosing the right inspection and repair intervals. Economically speaking this seems not a major issue if the overall uncertainties of cost estimates in early stages of the planning process are far larger.
In judging future environmental damage and in assessing the sustainability aspects of the design, maintenance and removal form however integral parts of the overall picture.

10.5 Economic life cycle approach

Switching from costs to the overall economic aspects of investment decisions in infrastructure, a few observations should be made. The theory of maintenance addresses without explicitly mentioning it, only the technical lifetime of the structures. Great effort is made to optimize maintenance policy in view of the cost of inspection and repair versus the expected cost of failure (risk) using the elegant tools of reliability theory. In the economic literature however a sharp distinction is made between this technical lifetime and the economic lifetime.
The limitations of maintenance theory become immediately clear if one realizes that there are two alternative solutions in case failure is imminent: 1) repair the old structure, 2) build a new structure. The second is omitted in the theory of maintenance, because it requires investment in a new structure.
In economic literature however much attention is given to this second decision, when old equipment should be exchanged for new. The simplest economic rule of thumb is that the full cost of the new structure should be smaller than the variable cost of the old. To show the difference with the technical approach a breakdown of the total cost is given for the two alternatives 'build new' and 'repair old' in next table.

Wages	Wages
Energy	Energy
Materials	Materials
Depreciation	--
Interest	--
Repair & inspection	Repair & inspection
"Risk"	"Risk"
Total Full cost "new"	**Total Variable cost "old"**

Table 10-4: Comparison of two solutions on costs

The entry "risk" is here given between quotes, because, although the driving factor in maintenance theory, it is not accounted for in traditional cost statements except for the cost of insurance.

Table 10-4 shows that the probabilistic maintenance theory optimizes only two entries in the right hand column **repair** and **risk** to keep the structure going. The economist is however also interested to know if the new structure will be more efficient and thus cheaper in wages, energy and consumable materials. If these cost improvements outweigh the depreciation and interest cost of the new structure he will propose to scrap or lay up the old structure although it is technically feasible to keep it going.
In some instances it is economically wise to look not only to the costs but also to the revenue because the new structure can also generate more income. If its service is more attractive to clients, their number and willingness to pay will grow. Therefore the economist generally prefers an improved rule of thumb that the profit "new" should exceed the profit "old".

Revenue "new"	Revenue "old"
Full cost "new"	Variable cost "old"
Profit "new"	**Profit "old"**

Table 10-5: Comparison of two solutions on profit

In the theory of maintenance the revenue generated by the structure is completely omitted. The theory assumes that the age and the state of maintenance of the structure have no influence on the level of revenue. Thus the variable 'revenue' vanishes as a constant in the derivative, if one tries to find the maximum profit mathematically. This is however generally not true in reality, that the revenue remains the same. Modern structures generate more income and require less labor and consumables to operate. The examples of technically perfect quay walls being abandoned for new berths with deeper draught, or well-maintained general cargo ports losing traffic to container ports, or well-kept ships being laid up in fjords show that there is more to the problem than refined theories for optimal maintenance.
Practically speaking the technical lifetime of most port structures exceeds the economic life time. It can even be stated that the real economic life time is often shorter than the planned economic life time during which the investment is depreciated. To avoid or mitigate this loss of investment several courses of action can be observed.
By intensifying the use of the structure the payback period is shortened and the surplus in strength is put to use. The 24 hour continuous (un)loading operation in modern ports is an example.
To make the assets movable is another idea. If the economic life time ends at one location the structure will be moved to another location where a new life starts. Floating ports are an example where the technical life time aims to span two or more economic lives. Movable assets make especially sense when political instability and nationalization threaten to shorten the economic life time from the beginning.
A third course of action is to assemble the structure of reusable parts. For instance a block wall composed of standard blocks that could be reused to build a new structure at e.g. a deeper berth or with a completely different function.
The last idea, that is difficult to apply, calls for structures with built-in flexibility. This is an attractive but rather general term that could well cover all ideas mentioned previously.
The two basic ideas of shortening the technical lifetime or extending the economic lifetime are only attractive if they make economic sense. So a shorter technical life should reduce the investment considerably to be economically viable. On the other hand it introduces the risk of interrupted operation if by lucky accident the economic lifetime might exceed the technical.
Also the additional investment needed to create the extra flexibility that could enhance the economic lifetime must be minor, because it is uncertain if it will ever be used.

10.6 Life Cycle approach to systems

The refined optimization of the design and maintenance of a single component-structure is put further into perspective by the fact that a successful port is a complicated system of many components. Simply stated a port is a series-system generally consisting of the following main components:

- deep water approach channel
- entrance channel
- breakwaters

- port lay out and handling vessels
- quays and jetties
- (un)loading equipment
- inland transport
- port management organization

In a series system the component with the smallest capacity limits the system capacity. This component is indicated as the limiting factor. In the same sense the reliability of the port depends on the component with the highest probability of failure.
First the capacity should be optimized. The component that requires the highest investment for a small increase in port revenue should be the limiting factor. All other components should have ampler capacity. Secondly the reliability of the port system must be limited by the component that requires the highest expense for maintenance to improve its failure probability. All other components that are cheaper to improve and maintain should have a higher reliability.
The optimization of the port system must be oriented at the revenue. The conceptual aim should be the maximization of the profit: the difference of revenue and cost, not minimizing the cost. In fact the port system should be optimized continuously, because the relative costs of the factors change incessantly. If wages rise or fuel costs increase another component might become the limiting factor with the highest marginal cost.

Frequently economics is called the dismal science, because it argues a causality that is contrary to popular belief. Ricardo (1821) showed that the high price of land was the consequence not the cause of the high price of corn. Following the same line of reasoning the siltation of the port of Bruges was the consequence not the cause of the economic decline. A successful port should be able to afford sufficient maintenance dredging.
So economically speaking there are a few trends that are certain to persist although many people see them as unpleasant and unexpected surprises. In real terms and excluding taxes

- Wages will rise
- Price of land will increase
- Price of commodities (incl. energy) will fall

These trends are the consequence of the growth of the world population and the desire to increase its welfare. More welfare for more people can only be reached by more production per man-hour and per hectare.

10.7 Conclusions

In this paper it has been shown that the theory of maintenance is an aid to life cycle management. The theory introduces the risk of failure as the driving force behind preventive maintenance strategies.
A serious limitation of the theory of maintenance is the omission of the renewal option. The theory aims to extend the technical lifetime of the structure by optimizing the inspection and repair intervals in view of the risk of failure. However one could also design and build a completely new structure. It appears this is central in the economic analysis of plant renewal.
Economic theory also points to revenue increases besides cost reduction in plant renewal decisions. It is clear that new opportunities are more important than optimal maintenance.
It is also explained that the optimization of a single component of a port system is less useful. A port is a series system consisting of many components. The capacity of the port will be limited by the component with the smallest capacity: the limiting factor. Similarly the reliability of the port will be limited by the component with the highest probability of failure. One should spend on the limiting factors until other components become limiting. In the optimal situation the limiting factors should be the most expensive to improve.
Finally it is observed that popular belief often confuses economic causes with consequences. So the perennial rise of wages is not an unfortunate phenomenon, but the aim of economic policy directed to increase welfare. In order to find bright ideas it helps to do some dismal thinking and question the traditional cause -> consequence reasoning by applying economic theory.

Was the siltation of the port of Bruges the cause or the consequence of the economic decline?

Suggestions for further reading

Vrijling, J.K., Smit, E.S.P., and De Swart, P.F., 1991. Berm breakwater design - the longshore transport case: a probabilistic approach, *Proc. Conf. on Breakwater Design and Construction*, Inst. of Civ. Eng., London.

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Vrijling, J.K., Smit, E.S.P., and De Swart, P.F., 1991. Berm breakwater design - the longshore transport case: a probabilistic approach, *Proc. Conf. on Breakwater Design and Construction*, Inst. of Civ. Eng., London.

Vrijling, J.K., and Van Gelder, P.H.A.J.M., 1996. Probabilistic Design of Berm Breakwaters, In Roger Cooke, Max Mendel, and Han Vrijling, editors, *Engineering Probabilistic Design and Maintenance for Flood Protection*, pages 181-197, Kluwer Academic Publishers.

CUR, 2004, Handbook of quay walls, CUR,Gouda

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11 Methods of risk analysis

11.1 General

It has been recognized by Markowitz (1952) that besides the height of the return, also the uncertainty of the return efficiency plays an important role in the appreciation of investment projects (Markowitz, 1952).
In literature, it is therefore often suggested to analyze the uncertainty of the yield of a project by means of a Monte Carlo analysis.

In this addition, the methods of risk analysis are presented which are used to analyze and approach the reliability of constructions in the field of engineering. These methods have shown to be very useful in examining risks of investments and such.

The easiest methods use trees, event trees and fault-trees, to examine an investment problem on a schematized system level.

In many cases, the black-white approach of these techniques is not sufficient and therefore the problem should rather be looked at as a function of continued variables. The calculation can be done with the help of probabilistic methods. As will be explained later, the exact methods which include complete integration and the Monte Carlo analysis, approximate methods are also used which generate good results very quickly.

The advantage of approaching methods is that, through the calculation, the relative contribution of each of the input variables will be specified according to the uncertainty of the project result.
With aimed measurements or additional research on the determined uncertainty sources, it is therefore possible to improve the appreciation of the project.

In this addition, first the theory will be discussed shortly which will be followed by an explanation by means of a simple example.

11.2 Risk analysis; event trees and fault-trees

A risk analysis consists of a couple of phases, which will all be discussed. During the first phase, a description of the problem as a system takes place. The required depth of the analysis will determine the degree of detail of the description. The other way around, a lack of information (for example because a project is still in its initial phase) restricts the depth of the risk analysis.

In the next phase, by means of brainstorming, one tries to gain insight in all possible undesired start events, which threaten a successful outcome of the project.
The normal financial analysis of the project as well as the experience gained from other projects might be helpful.

Subsequently, one investigates the possible reactions of the project on every undesired start event. The instrument used is the event tree, which logically links up between a start event and all possible following results of the project.

In the fourth phase, it is studied how the most undesired development of the project can occur. Here, actually, the most undesired branches of the event tree are united in a tree: the fault-tree. The most undesired development or event is placed at the top of the fault-tree: the top event.

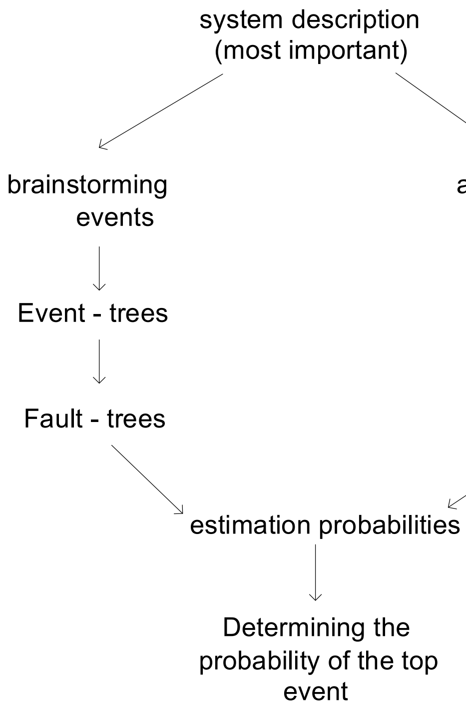

Figure 11-1: The development of a risk analysis.

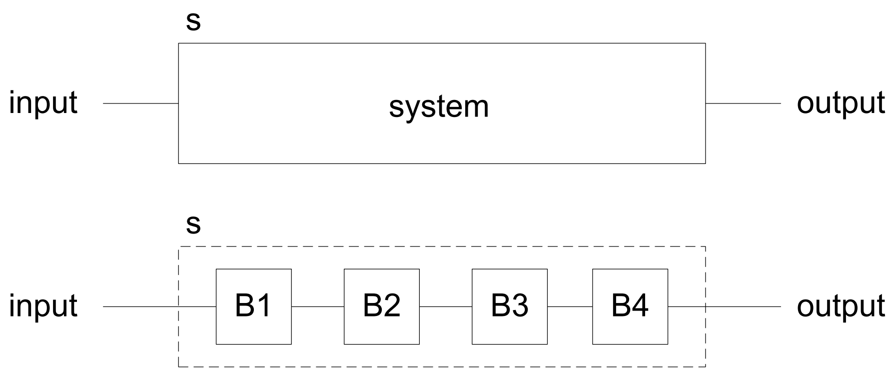

Figure 11-2: The division of a system in sub-systems.

So the fault-tree is a schematized description of the logical sequence of all events which lead to this top event (bankruptcy for example).
In case one wants to examine more than one undesired development, one should set up a fault-tree for every development.

In the final phase the chance of occurrence of the undesired top event is calculated. First the chances of occurrence of the start event are determined. This is done by experience (for example statistics) or by means of probabilistic calculations. Finally, according to conventions of the theory of probability, the probability of failure of the project is calculated.

An alternative method of working begins after the system description, with a systematic analysis of the system by separating it in series and parallel systems. Immediately afterwards, the fault-tree is set up. The rest of the execution is done in the same way as described above. (Figure 11-1)

The risk analysis uses the foundations of the system theory. The considered systems are described with input output blocks. Systems can be subdivided into subsystems, which in their turn can be divided into sub-subsystems. The practice usually gives a useful boundary for the depth of the division (Figure 11-2).

The two main ways of arranging the subsystems within a main system; the series and parallel connection are of great importance within the risk analysis. The difference is clearly shown when one tries to clarify the probability of failure with the help of event- and fault-trees.

The event – and fault-trees of a series system are shown in Figure 11-3.

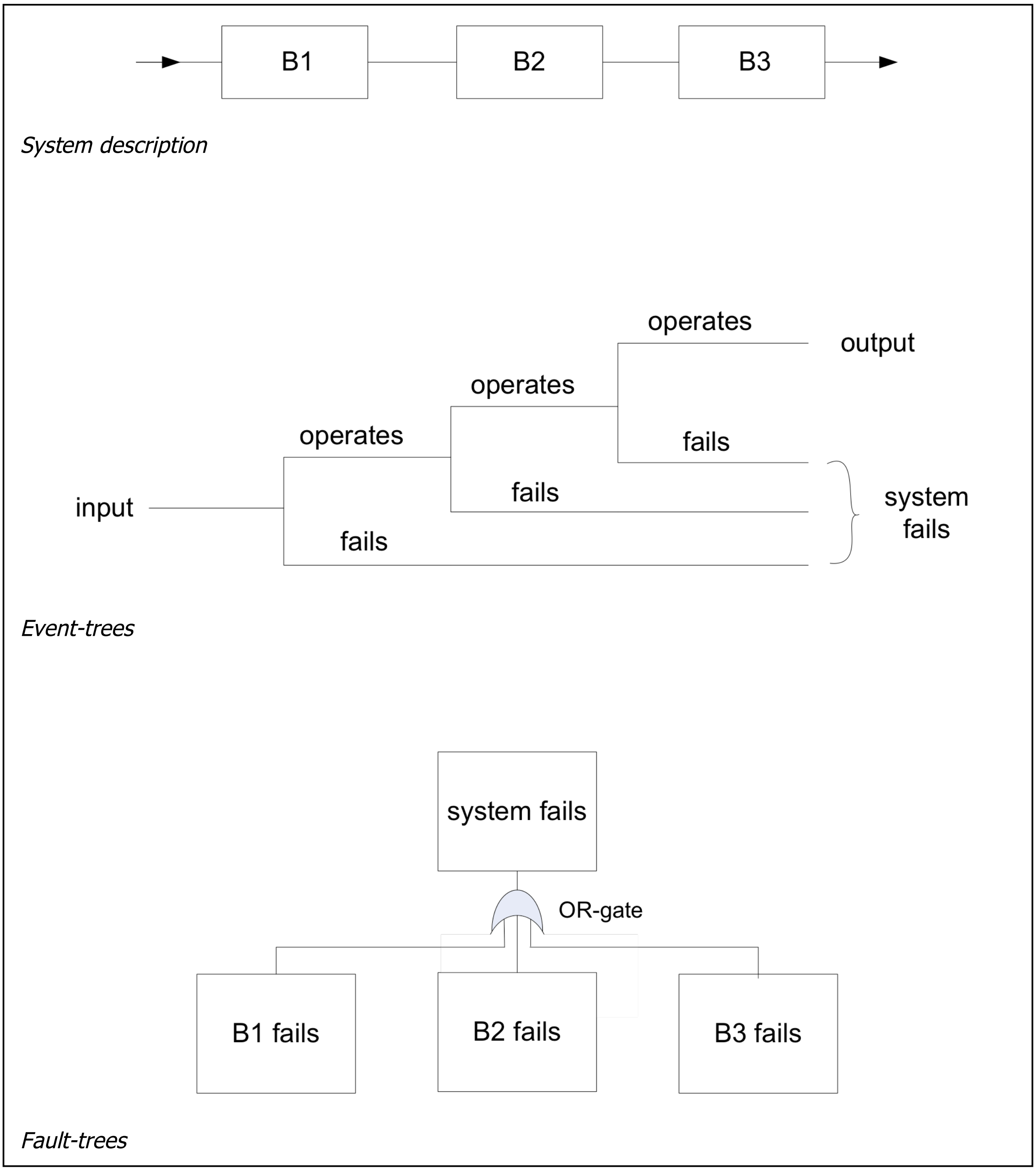

Figure 11-3: The risk analysis of a series system.

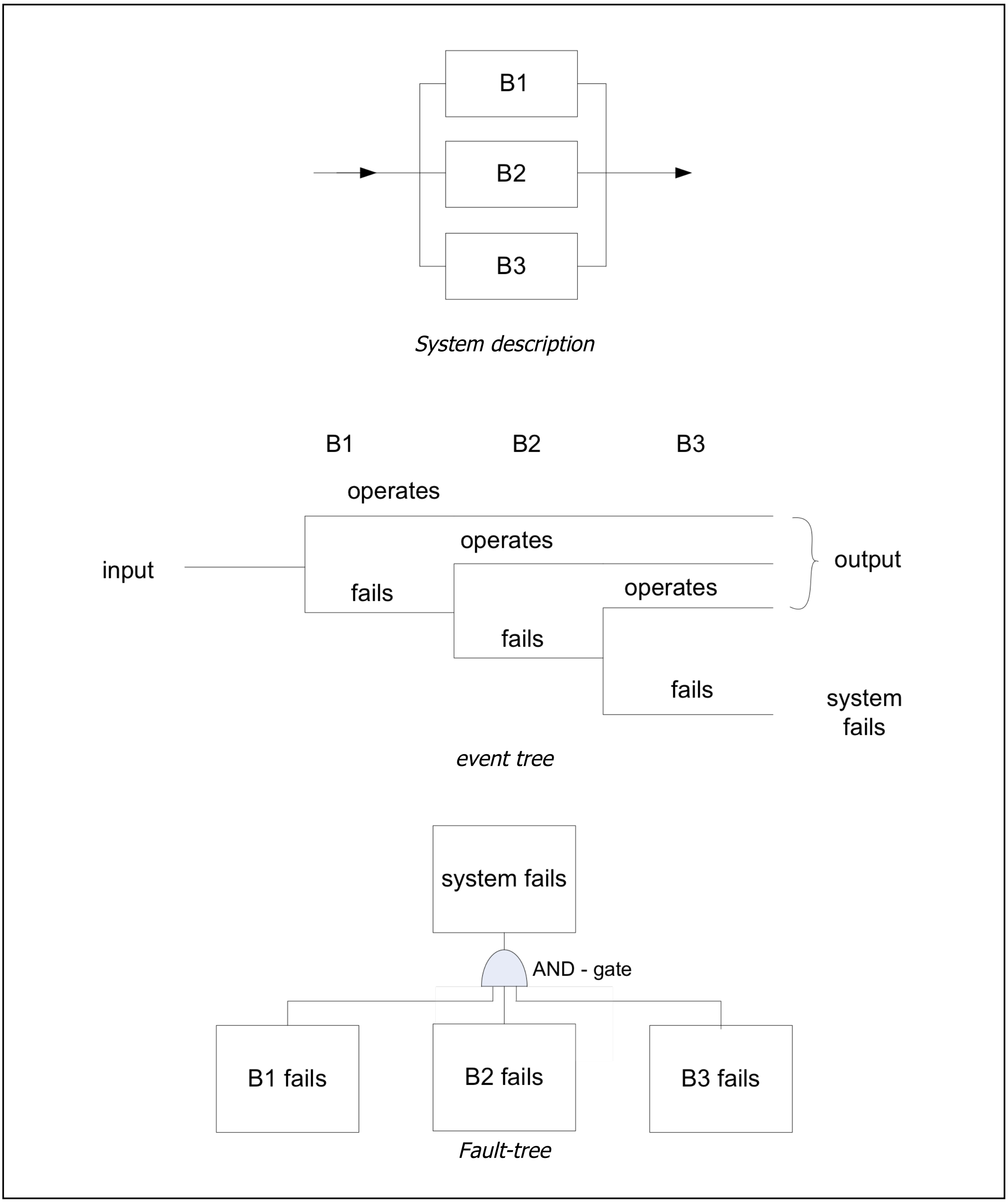

Figure 11-4: The risk analysis of a parallel system

One can see that the event-tree seeks for the consequences of a specific start event. This is indicated as 'forward logic'. It is common practice to put the 'unfavorable' events on the lower branch and the 'favorable' events on the higher branch in the event-tree. As expected, the failing of a single subsystem in a series connection is sufficient to bring down the whole system. In the fault-tree this is indicated with an OR-gate. The most unfavorable outcome is placed at the top of the tree; the start events form the base. The fault-tree is based on "backward-logic" and seeks for causes.

The probability of failure of the total system P(F) can be written as:
P(F)= P (B_1 or B_2 or B_3 or …. B_n)

In case the probability of failure of the base components is known and equal to P(B_i), the following holds for independency:

$$P(F) = 1 - \prod_{i=n}^{n}(1 - P(B_i))$$

$$\sum_{i=n}^{n} P(B_i) \text{ for small } B_i$$

The analysis of the parallel system follows the same lines. From the event-tree it follows that the system does not fail until all components of the system fail. This is indicated by an AND-gate in the fault-tree.
The probability of failure of the parallel system composed of n components holds:
P(F)= P(B_1 and B_2 and B_3 and…. B_n)

Or expressed as probability of failure of the components in case of independency:

$$P(F) = \prod_{i=1}^{n} P(B_i)$$

The result of the calculation clearly shows that a parallel system is of a safer order that the series connection.
The value of the risk analysis in the planning phase of a project is defined as follows:

- One gains an insight into the way a project can fail.
- One gains insight in the probability of failure.
- The trees function as communication tool and as tool of management.
- One can unify economical, technical and political causes of failing in one approach.

11.3 A simple case

The above described techniques can be demonstrated by means of a simple case. One considers the profit over a period of a company that sells a product. The costs per period are fixed.

The profit P over a period can easily be calculated by:

$$P = p * Q - K$$

in which:

p = selling price
Q = quantity
K = costs (= 90)

It is assumed is that the selling price and the quantity of the product are independent variables. Usually, the selling price is €1,25 and 80 pieces are sold. The chance that the selling price is lower than €1,125 is 0,16. There is also a chance of 16% that the sold quantity is less than 72 pieces. The costs K are given as a deterministic number.
In Figure 11-5 a schematized description of the company as a system is given. Besides, the event-tree and the fault-tree have been drawn.

The probability of failure of the company is as follows:

P(F) = P (B_1 or B_2)
= P (price <1,125> or (quantity <80>)
= 1 – (1 – P (price)) (1 – P (quantity))
= 1 – (1 – 0,16) (1 – 0,16) = 0,29
~ 0,16 + 0,16 = 0,32

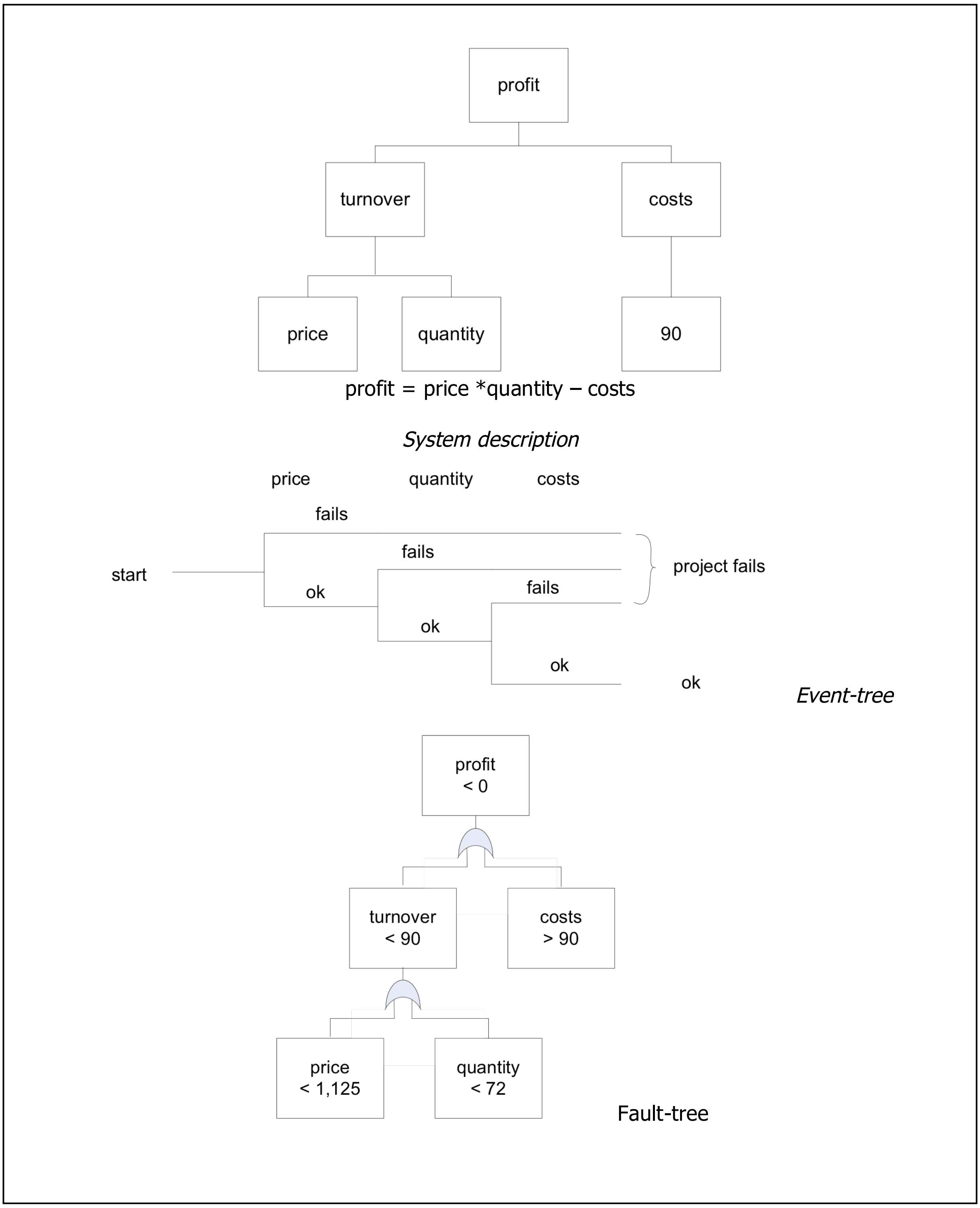

Figure 11-5: The risk analysis of a project.

The schematization of continuous variables such as price and quantity to bivalent variables (price = 1,25 or price <1,125>) may seem forced. After all, in real life are innumerable combinations of price and quantity imaginable of which only a fixed number lead to a loss (see also fig. 5.4).
The fault-tree technique, however, forces to such a schematization. The probabilistic calculation techniques, which will be dealt with later, do not have this disadvantage. In many cases, the risk analysis with help of the fault-tree does actually clarify the insight.

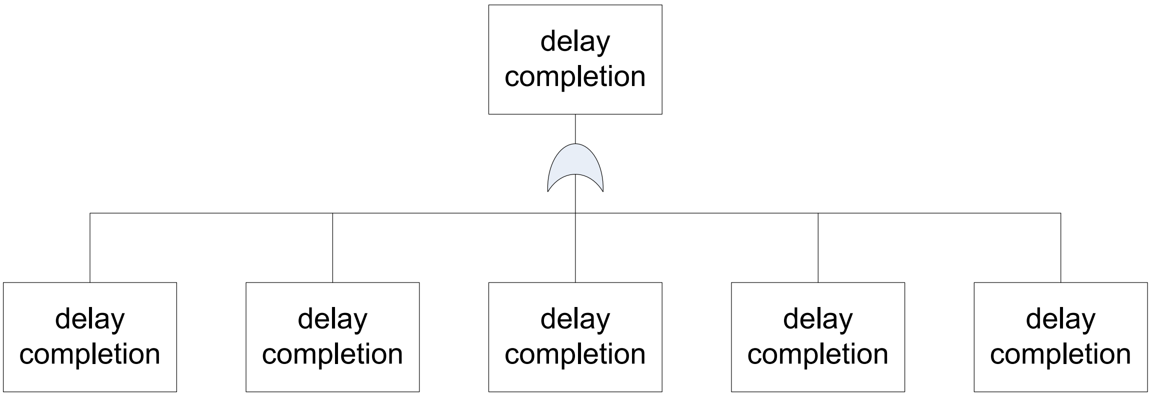

Figure 11-6: The fault-tree of the chance on delay on the completion of a construction.

In the figure above, an example of the practice is given.

11.4 An overview of the probabilistic theory

In the field of construction engineering relatively a lot of experience is obtained through the calculation of the chance a construction (for example a bridge or a dike) failing (Vrouwenvelder & Vrijling, 1984). This has created a useful set of instruments to examine systems which are a function of continuous variables.
The available methods for calculating the probability of failure of a continuous system are classified as follows:

Level III holds the exact probabilistic approach, in which the probability density function of all variables is taken into account. The Monte Carlo method and the method of total integration belong to this category.

Level II holds a certain number of approaching methods, in which the problem is linearized around a certain, carefully selected, point. Correlations between variables are to be avoided. Not normally divided variables need supplementary approaches. Because of the providing insight results and the speed of calculating, Level II methods are very popular.

Level I holds the normal design methods, which create a distance between the characteristic values of strength and loading by means of a set of partial safety factors.

The methods are formulated in such a general way that they can easily be used for problems of business economics. One can, for example, set a goal to calculate the probability on losses in a financial project.

The problem consists of two pieces.
First of all, there is the mathematical model which describes the course of events within a project.

In the probabilistic calculations, the model is formulated as a reliability function. In the simplest form it comes down to:
$P = I - K$
$Z = P$

in which:

P = profit
I = income
K = costs
Z = reliability function

The reliability function Z divides the I,K plane into two areas; the safe area (the profit area) and the unsafe or fail area (the loss area). The fail boundary lies between these two areas.

$Z > 0$ safe area
$Z = 0$ fail boundary
$Z < 0$ fail area

The project finds itself in a critical state in case $Z = 0$. (limit state)

The second part of the problem is formed by stochastic research. Suppose the probability density functions (PDF's) of the receipts I and the costs K are given by:

$$f_I(i) \qquad f_K(k)$$

In case the costs and the receipts are independently stochastic, the collective PDF is determined by:

$$f_{I,K}(i,k) = f_I(i) \ , f_K(k)$$

To find the solutions to this problem, the two pieces should be put together (see Figure 11-7).
In the I,K plane, both the fail boundary as well as the altitude pattern of the collective PDF (lines of equal chance density) are represented.

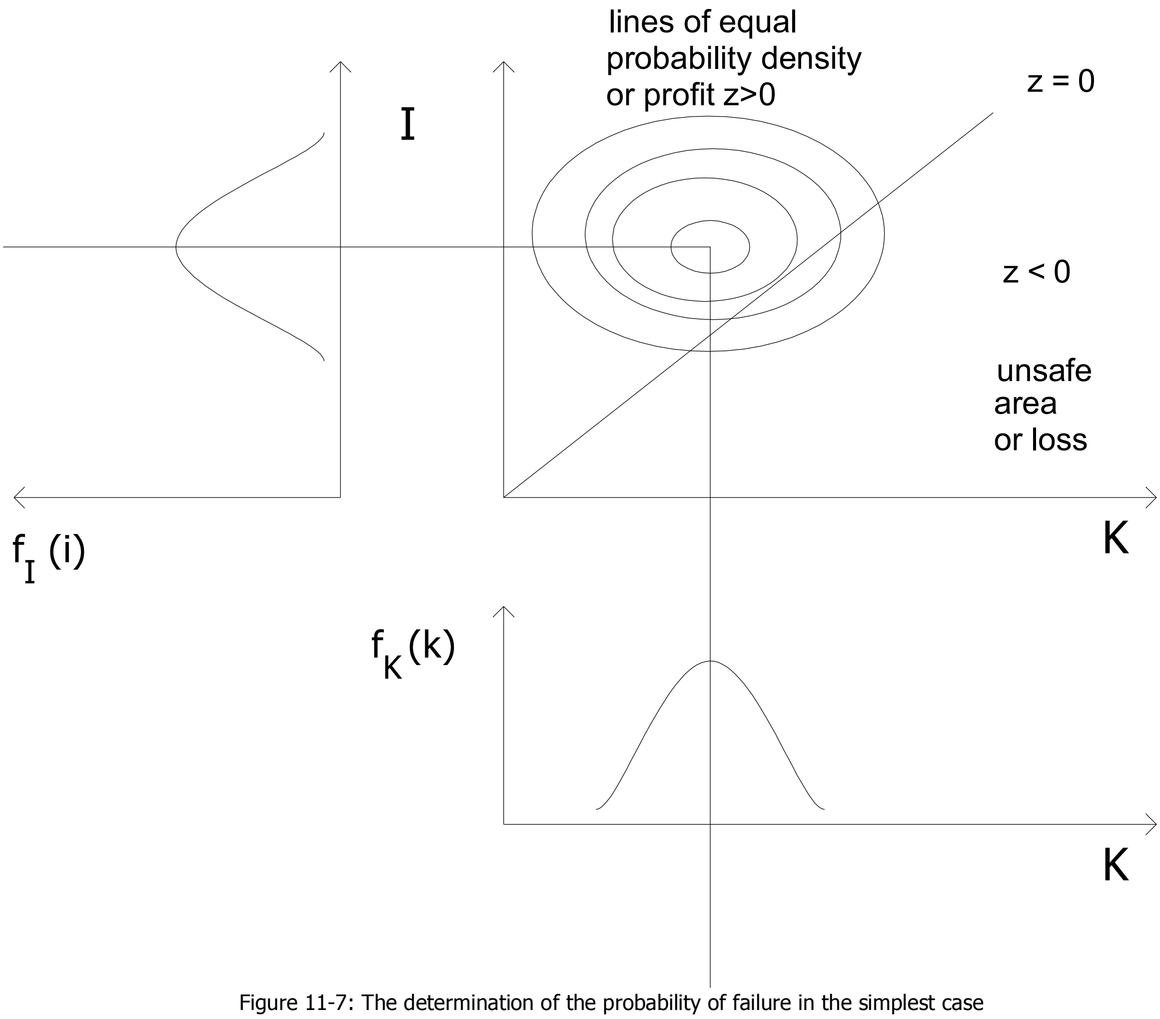

Figure 11-7: The determination of the probability of failure in the simplest case

It is easy to see that the probability of failure of a project is equal to the volume of the combined PDF in the area of failure (loss area).

In mathematical notation:

$$P_f = \iint_{Z<0} f_I(i) \quad f_K(k) \quad di \quad dk$$

The solution to this integral, where the business economics model of the project stands on the integration boundary, is in general only possible numerical.

The above mentioned method is also applicable when the income and the costs are known as functions. In a realistic case, the income and the costs are indeed a function of a certain number of stochastic variables.

Say the income is given as:

$$I = I\,(X_1\,,\, X_2\,,\, X_3 \ldots X_m)$$

And the costs as:

$$K = K\,(X_{m+1}\,,\, X_{m+2}\,,\, X_{m+3} \ldots X_n)$$

Then the profit is also fixed as a function of the variables and holds in this case the reliability function

$$Z = I - K = Z\,(\,X_1\,,\, X_2\,,\, X_3 \ldots X_n\,)$$

If, moreover, the PDF's of the stochastic variables are given, the combined PDF can be calculated easily in case of independency. The probability of failure follows from the multiple integral over the area Z < 0:

$$P_f = \iint_{Z<0} \ldots \int \; f_1(x) f_2(x) \ldots f_n(x)\, dx_1 dx_2 \ldots dx_n$$

This integral is easy to calculate numerically with the Riemann procedure.

The second level III method is the Monte Carlo simulation. This method, which is described in the standard literature of finance, is used a lot in the world of finance. The essence of the method is that, for all variables of the reliability function Z, one draws a value from the corresponding distributions. Subsequently, one calculates the value of the Z-function out of the drawn values of the variables. In case the value is negative, the project fails. By repeating the procedure several times, an estimation can be made of the probability of failure in the form of the quotient of the number of fail cases and the number of simulations N.

The disadvantage of the Monte Carlo method is that the outcome itself is a stochastic value with a mean and a standard deviation. It can be proved that the variation coefficient of the result decreases with the number of simulations and that it is conversely proportional with the probability of failure. Generally, the necessary number of simulations is given by:

$$N > \frac{C}{P_f}$$

In which C = Constant variable of the demanded accuracy

Thus, the number of variables which are part of the Z-function do not play any role. This contradicts with the solution by complete integration. The number of times that the Z-function should be calculated follows from:

$$N = s^n$$

In which: s = the number of integration steps that is expected to be the same for every variable.

In case the problem has a large amount of variables, the Monte Carlo procedure is the best way to minimize the time of calculation. Although if the main criterion is calculating the smallest probability of failure, the method of complete integration is preferred in case the computer is used.

In recent years, the new class of calculating methods, the level II methods, are being used a lot more. The reason is that, in comparison to the level III methods, they are practically applicable. Besides, they have the important advantage that they, as a by-product of the probability of failure calculation, give insight into the contribution of several variables with respect to the total uncertainty of the profit function. The level III methods give this insight only after additional calculations, which are in principle of the same proportions as the original calculation.

On level II, distinction is made between three sub-classes of methods:

1. Mean value – first order – second moment method
2. Refined – first order – second moment method
3. Methods which make use of the approaching probability distribution

The first version can be calculated manually easily and is therefore very instructive as a first approach. The level II methods are also based on the previously defined reliability function:

$$Z = I - K = Z(X_1, X_2, X_3....X_n)$$

The function can be linearized with help of a Taylor series expansion around a more specified point X.

$$Z_{lin} = Z(X_1', X_2', X_3'....X_n') + \sum_{j=1}^{n} \frac{\partial Z}{\partial X}(X_j - X_j')$$

An evident point to perform the linearization is the point that is determined by the average of the variables X. Of the by then linearized function, under the assumption of the independency of the variables, the mean and the standard deviation can be calculated easily, by:

$$mu(Z) = Z(mu(X_1), mu(X_2)...mu(X_n))$$

$$\sigma(Z) = \left\{ \sum_{j=1}^{n} \left[\frac{\partial Z}{\partial X} \sigma(X_j) \right]^2 \right\}^{0.5}$$

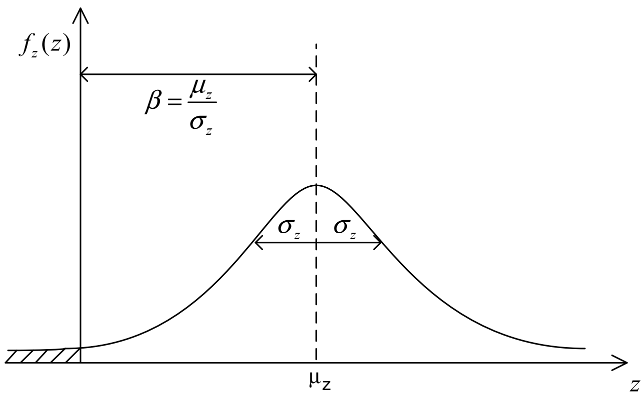

Figure 11-8: Determining the probability of failure with help of the reliability function

In case the variables are normally distributed, the probability of failure can be determined easily by the values of the reliability index beta (see Figure 11-8) and the table of the standard normal distribution:

$$beta = \frac{mu(Z)}{\sigma(Z)}$$

$$P_f = F_N(-beta)$$

in which F_N = standard normal distribution

The different sub-classes of level II are not discussed any further. The interested reader can read about it in literature [2] or [3] (in Dutch).

The final class, level I, contains the (in engineering) daily used design methods. In general this comes down to a defined minimum requirement of the value of the following quotient:

$$gamma = \frac{I}{K}$$

In which:

gamma = safety coefficient

In several fields of work there are minimum values for the safety coefficient, based on experience but also on the calculations of the probability of failure.

11.5 The simple case

The above described calculation methods will be presented with help of the in paragraph 3 mentioned simple case. To be able to explain the problem, the number of variables is restricted to two.

P = p Q – K
Z = p

The crucial failure boundary Z = 0 is given in fig. 5.2 in the p, Q – plane. The selling price and the quantity are assumed to be continues variables. Both are independent and normally distributed. The costs K are given as a deterministic number. The distribution parameter for the first two variables is given in the following table

	mu	σ
P	1,25	0,125
Q	80,00	8,00

The probability distribution functions are drawn separately in Figure 11-9 because the price and the quantity are assumed to be independent, the mutual PDF is calculated by:

$$f_{p,Q}(p,Q) = f_p(p) * f_Q(Q)$$

The contour lines of this two-dimensional PDF and the failure boundary are drawn in Figure 11-11 and Figure 11-10.

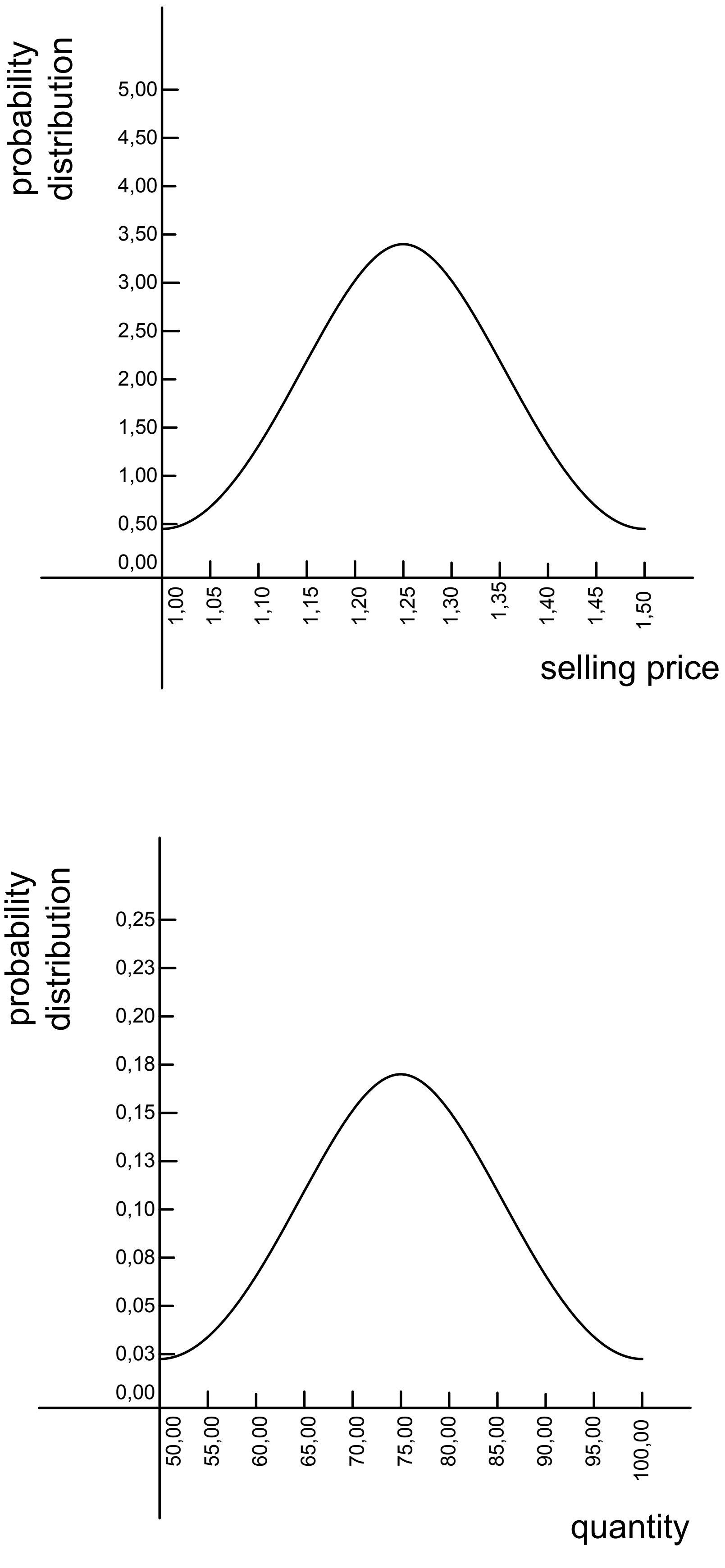

Figure 11-9: The PDF’s of price and quantity

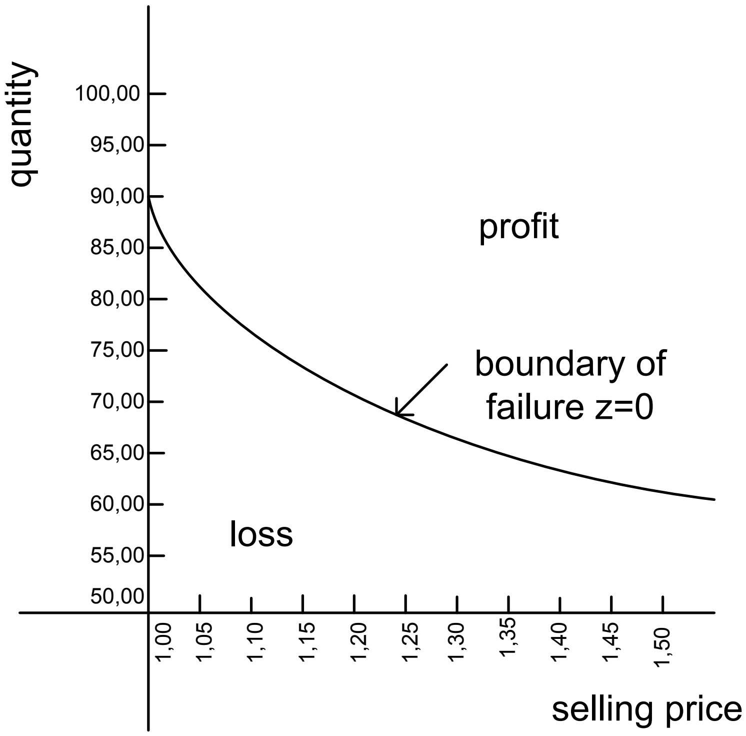

Figure 11-10: the boundary of failure z = 0

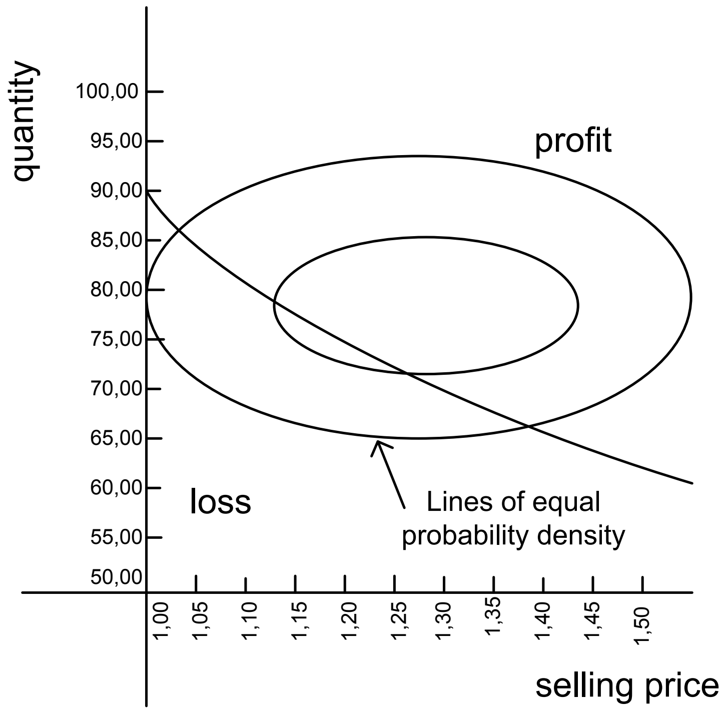

Figure 11-11: The two dimensional PDF of price and quantity

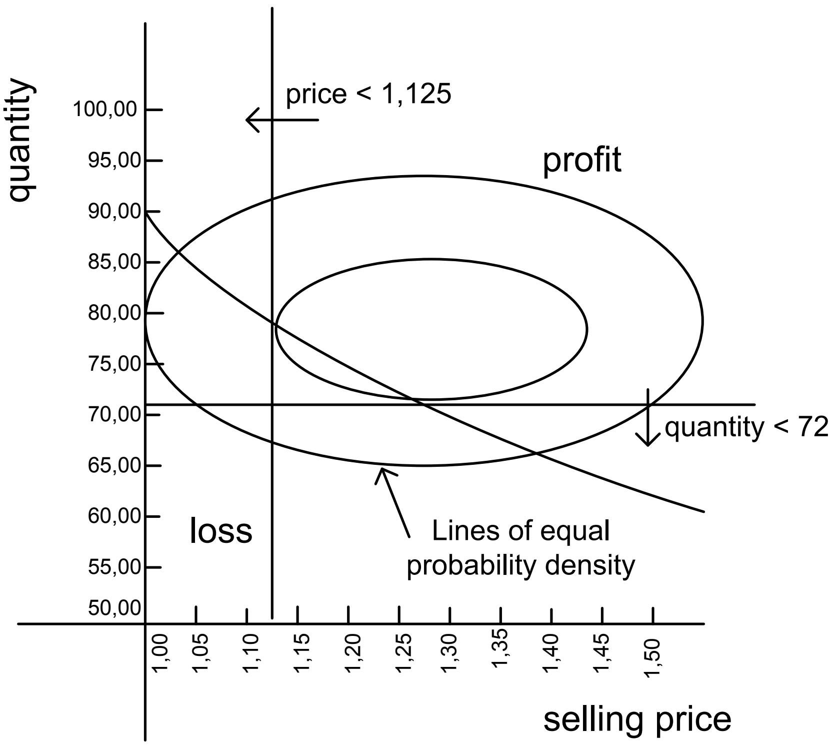

Figure 11-12: Calculation with help of the fault tree

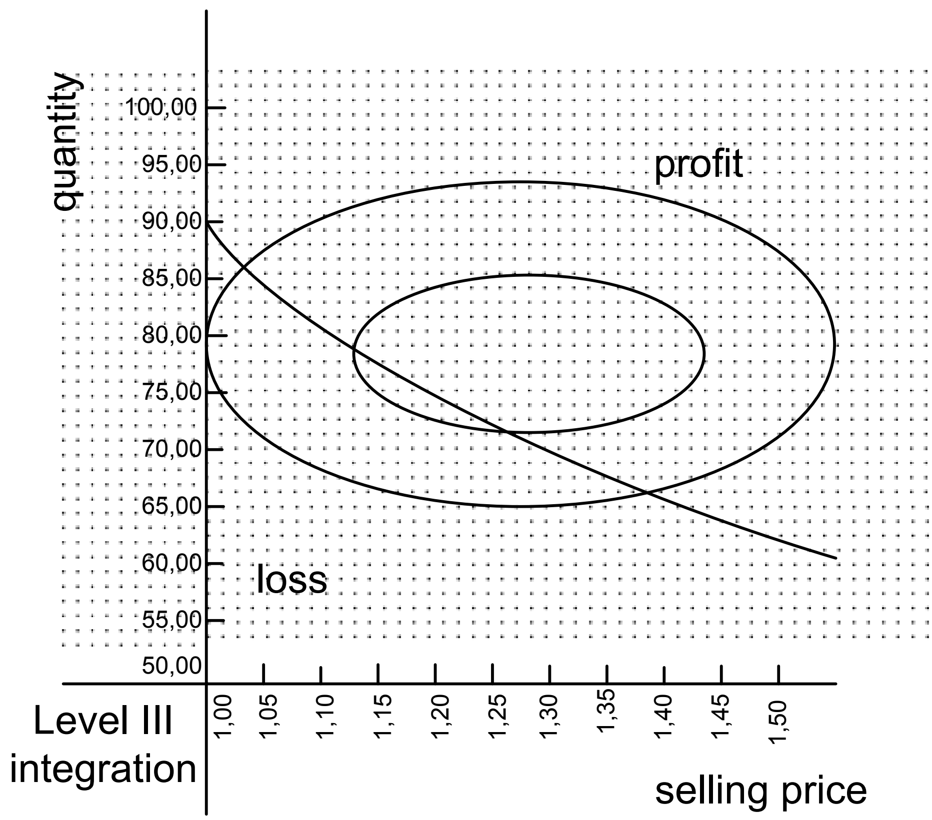

Figure 11-13: Calculation by means of total integration

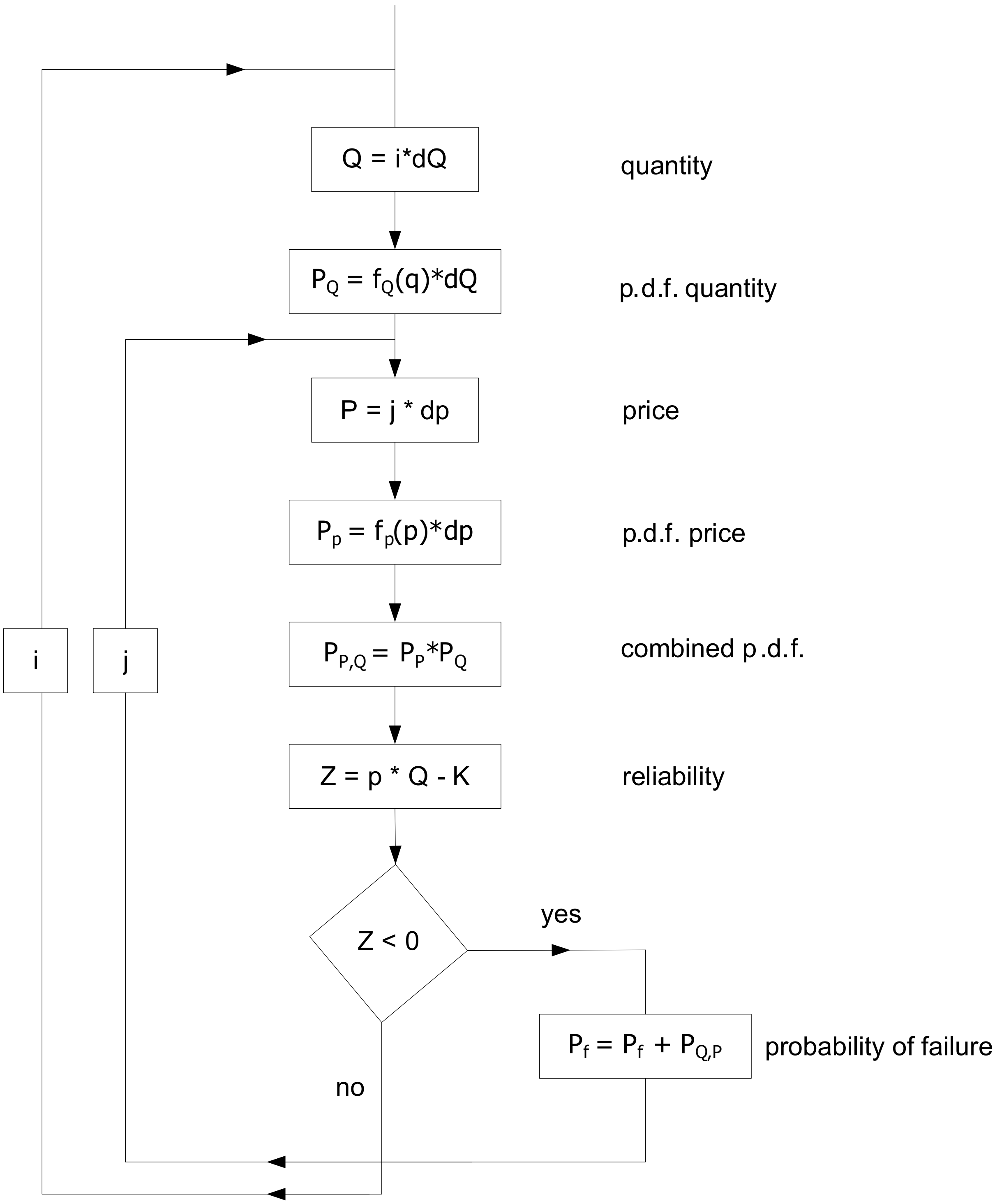

Figure 11-14: The flowchart for integration

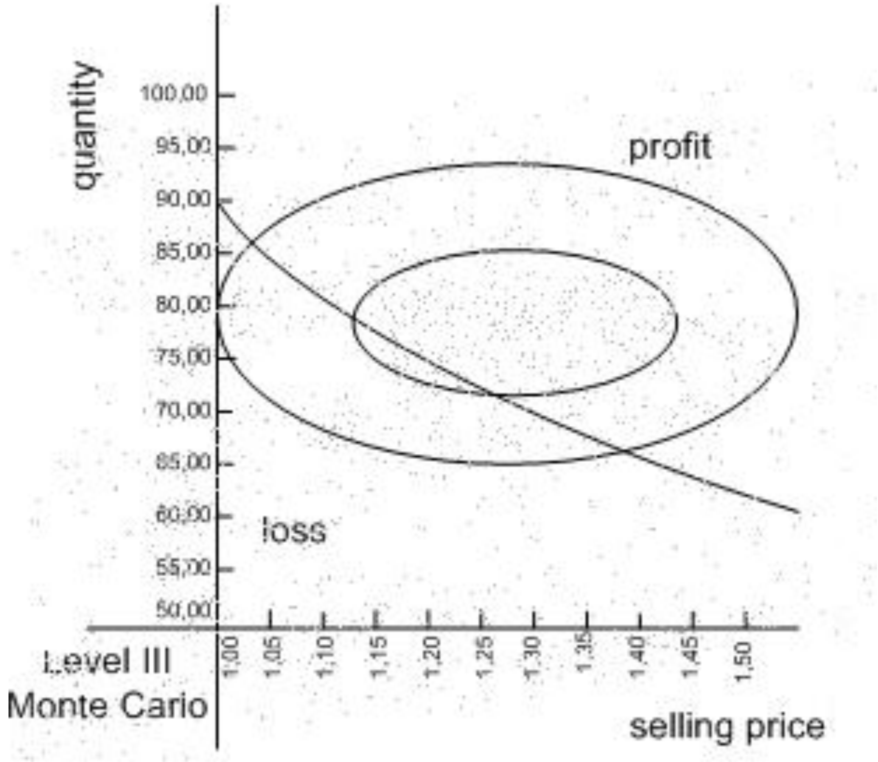

Figure 11-15: The Monte Carlo analysis

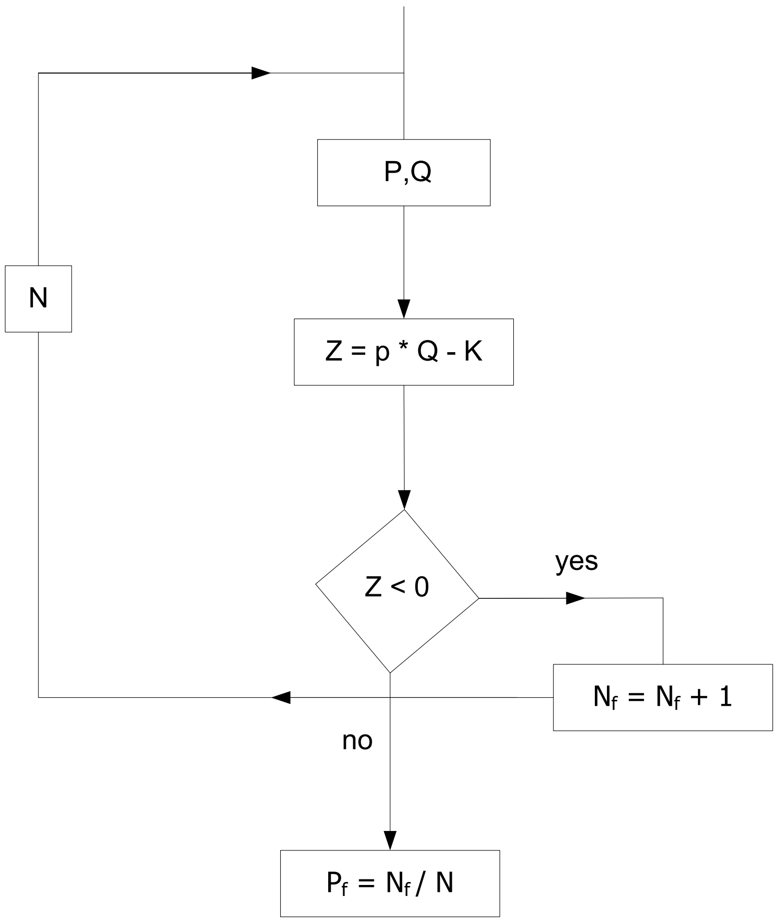

Figure 11-16: Flowchart for the Monte Carlo analysis

The probability of failure follows from the integral:

$$P(F) = \iint_{Z<0} f_p(p) * f_Q(Q) * dp * dQ$$

Numerically the integration takes place by systematically going through the whole p,Q-plane and by summing up the partial probabilities as long as Z < 0. The flowchart of the calculation is given in **Error! Reference source not found.** and progress of the calculation is plotted in Figure 11-13. The calculated probability of failure is 0.247.

The second level III method, the Monte Carlo Method, can also only be calculated with help of the computer. With this method, the value for p and Q are drawn out of the respectively distributions several times. In case of the drawn combination leading to a loss, it is a case of failure. From the ratio between the number of cases of failure and the number of situations N, an approximation of the probability of failure follows.
The flowchart of the calculation is given in Figure 11-16 and a plot of the progress of the calculation in the p,Q-plane can be found in Figure 11-15.
To show the stochastic character of the outcome of the Monte Carlo analysis, the results are presented below.

N	P(F)
900	0.24
900	0.27
900	0.26
10000	0.249

The mean value variant of the level II methods can be easily calculated by hand. The mean value Z can be calculated with the help of the mean values of the two basic variables price and quantity:

mu (Z) = 1.25 * 80 - 90 = 10

The calculation of the variance of Z is done in the form of a table:

	σ	dZ/dX	dZ/dX*sin(X)	(dZ/dX σ(X))^2	Alfa^2
p	0.125	Q = 80	10	100	0.5
Q	8.00	p = 1.25	10	100 +	0.5 +
				var (Z) = 200	1.0
				σ(Z) = 14.1	

The last column of the table shows the relative contribution of every variable to the variance of Z. The reliability index and the corresponding probability of failure are:

$$beta = \frac{mu(Z)}{\sigma(Z)} = \frac{10}{14.1} = 0.707$$

$$p_f = F_N(-0.707) = 0.24$$

With an average profit percentage of 10%, it turns out that according to the mean value variant of the level II methods, the probability of failure is 24%. This corresponds very well with the earlier calculated value with the integration method of the Monte Carlo analysis.

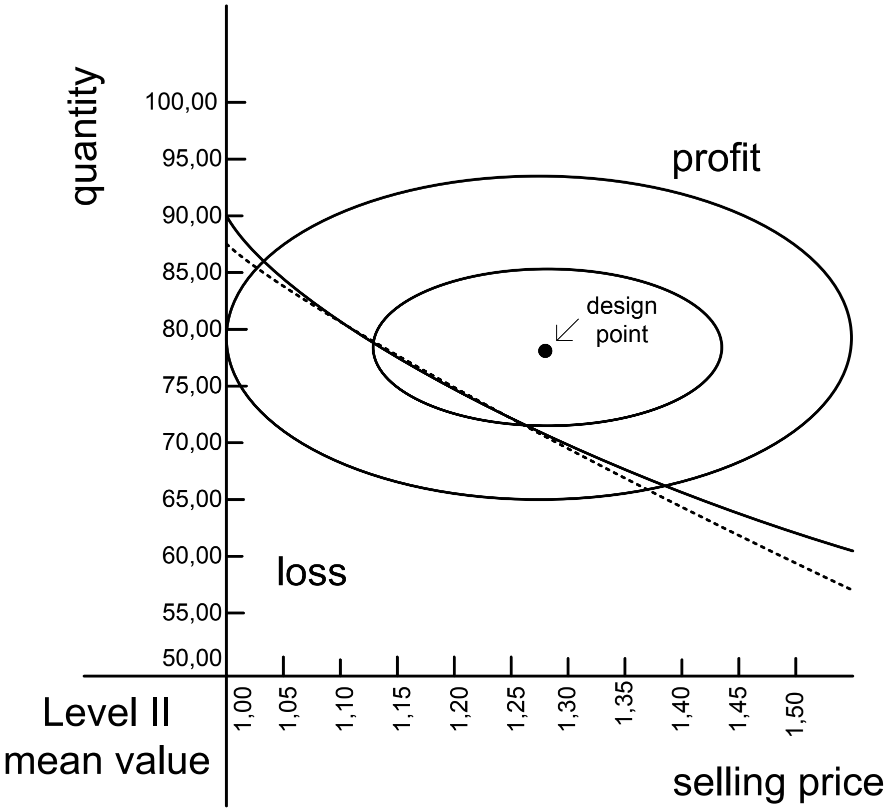

Figure 11-17: Level II mean value

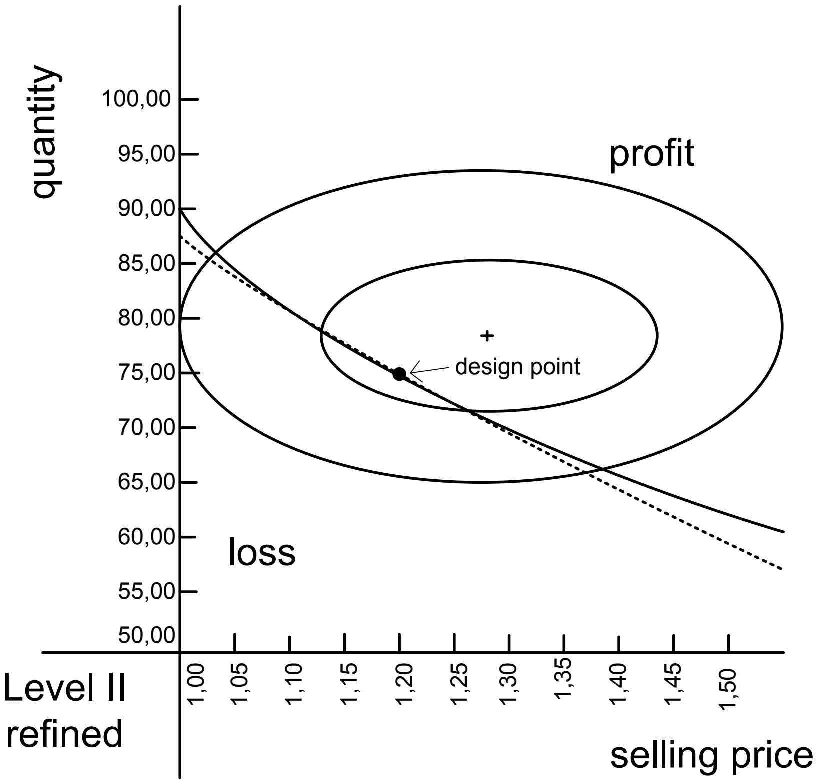

Figure 11-18: Level II refined

The table shows why the level II method is also called a 'weighted sensitivity analyses'. The sensibility of the problem of fluctuations in a variable is measured by means of the first derivative, while the standard deviation is a measurement for the possible fluctuations. The multiplication of both is a weighed sensitivity.

To be able to see what the approximation method holds, both the exact reliability function Z=0 and the linearized version are drawn in Figure 11-17. It is clearly visible that because of the linearization the failure zone at the flanks is being underestimated. When one determines exactly the probability of failure with help of complete integration, the result is also 24.7%.

Beta = 0.726
Probability of failure = 2.3E-0001

Name	Type	Mu	σ	X'	Alfa²	dZ/dX	X dZ/dX
Price	N	1.250	0.125	1.186	0.500	75.895	90.00
Quantity	N	80.000	8.000	75.895	0.500	1.186	90.00

Z(X) = 0.0000

Number of iterations = 4 Time of calculation = 0.10 s.

Table 11-1: Output of the refined level II calculation

On level II there are however more refined methods available. The improvement consists of a linearization around the point Z=0 with the highest density. The integration error will be at a minimum (see Figure 11-18). In literature [2] there are particular efficient integration methods available to find point X' with the highest probability density. Subsequently the mean and standard deviation of Z will be determined. Finally the probability of failure can be found through the reliability index, on the same way as with the mean value variant. In the table above the result of such a calculation of the simple case is given. In the fifth column the coordinates of the point with the highest probability density X' is given. In the next column the contribution of the variables to the variation of Z is given.

Finally it is interesting to examine what happens with the level of the probability of failure that when lowering the costs the benefits/costs ratio (or the percentage of profit) rises. Such a ratio is more or less the counterpart of the safety coefficient used in engineering.
For the present example, the relation between the benefits/costs ratio and the probability of failure is drawn in Figure 11-19. This relation is not generally valid. This relation can only give an indication in case of similar projects with the same structure and comparable uncertainties.

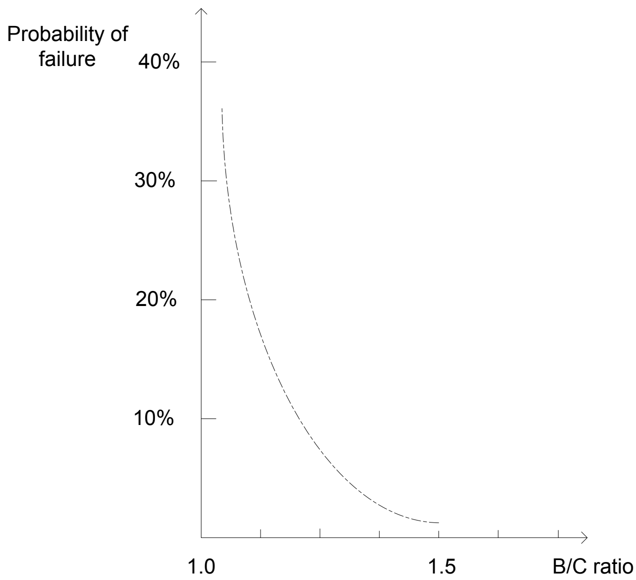

Figure 11-19: The relationship between the benefits/costs ratio and the probability of failure of the project

11.6 Summary and conclusions

In this contribution an overview is given of the methods, which are available to calculate the probabilities of failure of systems. So far as known only one of these methods are used in business economics, namely the Monte Carlo method.

It was stated in which way we can use the fault- and event trees with respect to risk analysis and projects. These methods increase the insight and are particularly suitable in approaching the different causes of failure. However, when looked at a more in depth analysis, the discrete character is a disadvantage.

For problems with continue variables, the non-probabilistic (aprobabilistisch) calculation methods are more suitable. The well-known Monte Carlo method and the method of total integration are part of the highest class of exact methods (level III).

The last few years, however, a lot has been calculated with the approximation methods (level II) in the field of engineering. The advantage of these methods is, besides their quickness, that they give insight into the contributions of the different stochastic variables to the variance of the end result.

By means of a simple case it was shown how the mean value variant of level II methods is calculated by hand and what principle it is based on. Subsequently the simple case was calculated with the refined variant and the differences were illustrated.

The probabilities of failure give an impression of the risks of the project and the weighted sensibility are of particular value in determining priorities in further analysis of the project. It was also indicated that there is a relationship between the benefits/costs ratio and the probability of failure of the managed project. Although this relationship is far from being generally valid, it can globally shape a guideline for similar projects. In fact one applies the level I method by setting a minimum requirement for the benefit/cost ratio like it is a safety coefficient.

Suggestions for further reading

[1] Markowitz, H., Portfolio Selection, J. of Finance, March 1952
[2] Werkgroep 10, T.A.W., Probabilistisch ontwerpen van waterkringen, Delft, October 1985

11.7 Probabilistic Budgeting and Time-Planning

Authors: J.K. Vrijling & P.H.A.J.M. van Gelder, Delft University of Technology, Delft

Abstract: In this paper an overview is given of the principles of probabilistic budgeting and time planning. Uncertainties related to normal – and special events are described. Analytical expressions are presented. To deal with correlations between special events, an alternative for the classical product moment correlation coefficient is proposed.

11.7.1 Introduction

The management and control of the costs and the duration of civil engineering projects have been studied in literature since the early 1970's [5, 3, 7, 8]. Apart from uncertainties concerning quantities and production times (such as unit prices and wages), uncertainties involving economics and "influences from the outside world" (essentially changes in the design of the project), the difficulties in modeling the budgeting and time-planning are caused by the long period over which a civil engineering project is stretched. This period starts when the social demand for change is felt and the first plan for the project is outlined. The end of the period can be set at the delivery of the final product and the settlement of the bill.

The estimate of the budget is an approximation of the real costs. If all knowledge and facts have to be expressed in one single number, as is often required, discrepancies between the estimate and the finally realized costs cannot be avoided. Normally, in comparisons between the original estimate of the building costs and the total expenses at the end of the project, no correction is incorporated for the overall increase in prices. In Table 1, the exceedance of the budgets for the hydraulic engineering projects of the reclamation of some IJsselmeerpolders is given [1]:

Table 1:

Polder:	Exceedance
Wieringermeerpolder	12%
Northeast polder	16%
Easterly Flevoland	-3%

The difference in accuracy between an estimate of the budget in an early stage of the project (the study-of-plan phase) and the final estimate (builder's specifications at the start of the engineering) is illustrated in the next table [3]:

Table 2:

Project	Difference in % of the final costs	
	Estimate in study-of-plan phase	Estimate in Builder's specifications phase
Haringvliet locks	77%	22%
Grevelingen dam	-19%	22%
Volkerak dam	56%	23%
Brouwers dam	-39%	-18%

Often final project costs exceed their estimate. Historical estimates of the budget at the Ministry of Public Works in The Netherlands (Rijkswaterstaat) [4] clearly show a pattern of increase of costs and delay in work (which, due to inflation and loss of interest, also increases the costs):

Table 3:

Project	Start	Planned		Reality	
		Mf	years	Mf	years
Noordhollands Canal	1818	4	5	12,5	7
Haarlemmer lake	1837	8,4	5 (?)	13,8	21
Nieuwe Waterweg	1858	5	5	36,8	38
Maas & Waal	1864	6,5	10	24,7	44

In road-construction projects there were (and there are) large fluctuations in the differences between the estimated budgets and the real costs as well. For road-construction projects in The Netherlands from 1980 up to 1985, differences between estimates in the pre-design phase and the real costs as a percentage of the real costs are given in the following histogram:

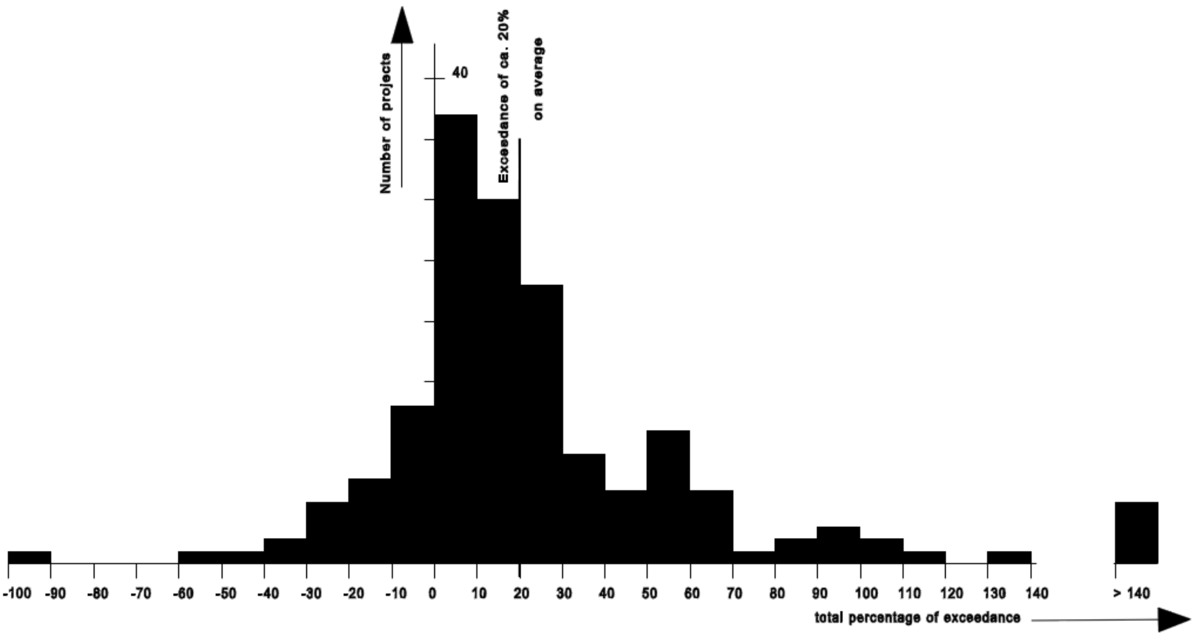

Figure 11-20: Differences in cost estimates and real costs

Publications on exceedance of the estimates of budgets in other countries show the same tendency [2] and [5].

11.7.2 The classical approach to budget estimates and time-planning

From early days, the calculation of estimates of the budget and time-planning schemes are based on the most likely values. Uncertainty of the decision maker is expressed as an increase of the deterministic final amount by a certain percentage of that amount.

Uncertainty regarding the budget estimate or the time-planning scheme is not constant during the project. The later on in the project an estimate of the budget or a time-planning scheme is made, the more about the project is known and the decision maker's uncertainty concerning the estimated amounts of money and duration of the activities will be less than in the early stages of the project. A classification of project phases in order of time is given in Table 4. The project parts in the phase at hand are estimated or planned in detail, for other parts in other phases the estimates are determined roughly.

Table 4:

Class	Project phase
D	study-of-plan
C	pre-design
B	pre-builder's specifications
A	builder's specifications

Because of the decision maker's greater uncertainty in the early phases of the project, it is of no use to make detailed estimates and time-planning schemes for phases to come. Making an estimate of the budget in detail for builder's specifications when the project is still in the study-of-plan phase will turn out to be a waste of time, although in the early stages more detailed estimates and time-planning schemes (or parts of those) are made.

Budget-estimates

First, some examples of specifications of budget estimates in several phases of a project are given.
Estimate of the budget, Class D (Study-of-plan phase)

1 viaduct × price of 1 viaduct = item viaduct
5 km of road × price per km of road = item road
1 tunnel × price per tunnel = item tunnel
------------------------ +
Total of Direct costs
Indirect costs
------------------------ +
Primary costs
Additional costs
Miscellaneous[4])
------------------------ +
Basic estimate
Unforeseen [5])
------------------------ +
Estimate (ex. VAT)
VAT
------------------------ +
Study-of-plan phase estimate

In the successive phases the items of the class D- estimate are worked out in more detail. An example is given in the following class B- estimate.

Estimate of the budget, class B (Pre-builders specifications phase):

800 m3 soil × price per m2 = item "soil"
800 m3 concrete × price per m3 = item "concrete"
1 ton reinforcement × price per ton = item "reinforcement"
800 m2 formwork × price per m2 = item "formwork"
---------------- +
Subtotal viaduct

The other items are detailed analogously.

---

[4] In "Miscellaneous" those costs are categorized which are known but which are not specified. For a study-of-plan phase estimate these could be: land (has to be bought) preparation, deflection of conduit-pipes and water courses, temporary diversion of traffic, etc.
[5] "Unforeseen" is taken as an additional percentage of the Basic estimate here. If there is little insight in the character of the item Unforeseen then this way of calculation is applicable.

In a class A-estimate (estimate for builders specifications) the prices per m^3, m^2 or ton are determined in more detail, based on working methods and quantities. In an estimate in this phase, time and equipment are taken into consideration. Contractors prefer this method of estimating.

A sub-item of the item SOIL of the partial project ROAD from the class D-estimate (5 km of road) is chosen as an example of specification of an item in a class A- estimate:

For the delivery and processing of 80,000 m3 of soil for the partial project "road" the costs of the following means of production are estimated:

Delivery at the quay by ship	*80000 m3*	×	*price per m3*	=	*partial item 1*
Lease of an unloading plant	*80 days*	×	*day tariff*	=	*partial item 2*
Transport to location (by cars)	*800 days*	×	*day tariff*	=	*partial item 3*
Equipment for processing and compaction	*85 days*	×	*day tariff*	=	*partial item 4*
					---------- +
					Subtotal soil

The price per m^3 of processed soil is calculated by division by the volume in m3 (here: 80,000).

In principle, the estimate of Direct costs (an example of which was given for a class D-estimate at the bottom of the previous page) follows from an addition over all N partial items of the multiplication of quantity, hi, and the prices per unit, pi (see the calculation scheme of the budget estimate on the next page). Indirect costs can be calculated by an additional percentage, %1, which is a fixed percentage of the Direct costs.

Additional costs and Miscellaneous can both be expressed as a percentage %2 of the Primary costs. As stated above, additional costs are established as a percentage of the preceding part of the estimate. The Total estimate can thus be expressed as a function of the total of the Direct costs. A percentage of (the part of) the afore calculated estimate is called an additional percentage[6]).

In any phase of the project such percentages can differ. Generally, the Total estimate, in which the additional percentages are included, is calculated from the total Direct costs:

$$Estimate = \left(\prod_{j=1}^{M}(1+\%)_j\right)\left(\sum_{i=1}^{N} h_i p_i\right)$$

in which:

M = the number of additional percentages
$\%j$= the j^{th} addition over the foregoing subtotal
N = the number of cost items in the Direct costs
h_i = the quantity of the i^{th} item in the Direct costs
p_i = de unit price in the i^{th} item in the Direct costs.

Time-planning

In an estimate of the budget the unit prices have to be multiplied by their quantity and then added. In a time-planning only the duration or the lengths of time of all activities have to be added and no multiplication is needed:

[6] An additional percentage (for example the percentage Unforeseen) can be seen as a random variable or as a deterministic constant. For example the percentage VAT was fixed at 17.5% for a long time. It can be regarded as a fixed constant, unles it is expected to be changed in the future. Then, introducing the percentage VAT as a random variable is an option. In 2009, VAT is 19%

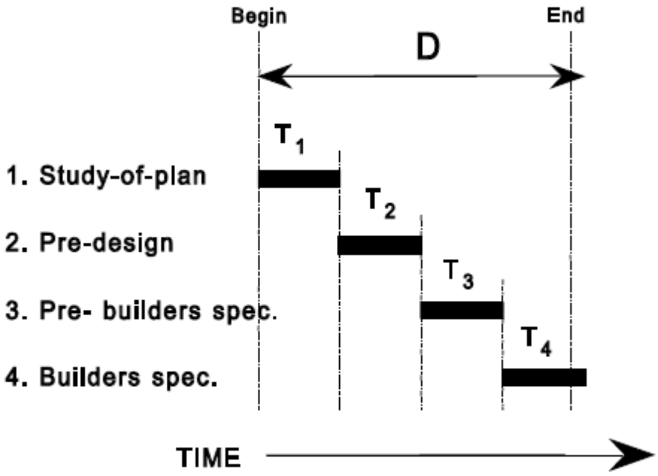

Figure 11-21: Time planning

The duration of the project, D, equals:

$$D = \left(\sum_{i=1}^{N} T_i \right)$$

in which:

> *i* = rotation number of the activity
> *Ti* = duration of the activity i
> *N* = number of activities (in Figure 11-21: N = 4).

If the various activities succeed each other in time the time planning is simple. Example: In a small building pit, pile driving cannot be started before digging of the pit is completed. The digging hinders pile driving too much.
Usually not all activities succeed each other in time. Various activities are (partially or totally) executed simultaneously. A consecutive activity can often only be started when more than one preceding activity have been completed.

In the figure below it is shown that the pre-builder's specifications phase can only be started when the pre-design phase has been completed and the licenses have been granted. Both the activities have to be completed before the pre-builder's specifications phase can be started. If two activities are executed in parallel (in Figure 11-22: the pre-design phase and the granting of licenses) the time-planning can be sketched as follows:

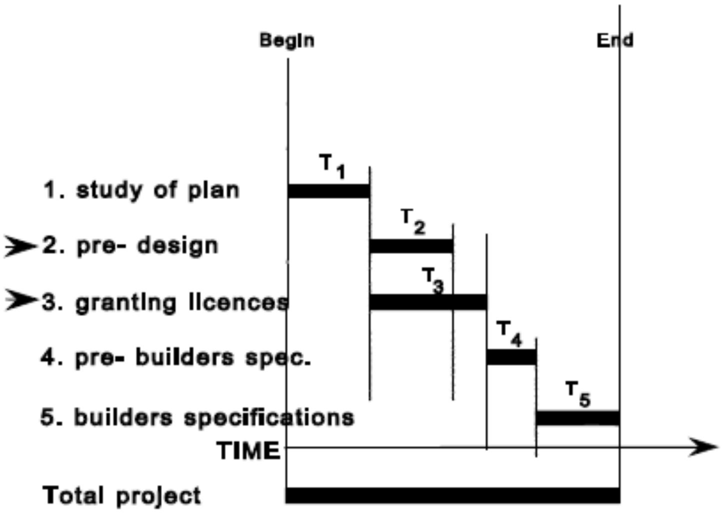

Figure 11-22: Project time planning

In this example there are two time-paths:

$$D_a = T_1 + T_2 + T_3 + T_4 + T_5$$
$$D_b = T_1 + T_2 + T_3 + T_4 + T_5$$

The total duration of the project, Dt_{ot}, is the maximum amount of time according to the duration of the various time-paths: $D_{tot} = \max{(D_a, D_b)}$
In the example in Figure 11-22, the duration of the activities 1, 3, 4 and 5 determine the duration of the total project, D_{tot}. It is said that these activities form the *critical time-path* (critical path for short).

11.7.3 Uncertainty concerning budget estimates and time-planning
In order to express one's uncertainty about the estimate or the time-planning, probabilistic techniques are employed. If the random character of the cost items or the duration of the various activities are taken into account, the budget estimate or time-planning of the project is said to be statistically controlled.
The estimated amount of money or the planned duration of a project can be interpreted in various ways, depending on what the person who estimates or plans has in mind. Does he or she focus on an estimate of the mean costs or duration (the expected values) or on the amount that is most likely (the mode of the costs or the duration)? In the first case it is commonly assumed that the costs or the duration are normally distributed. The mean and the mode then coincide. In the second case other (skewed) probability density functions are possible. Triangular probability density functions are often used for this purpose.

In addition to the mode or the mean of the estimate of the budget or the time-planning, the deviation from it (or the spread around it) is important. The size of the margin depends on the phase the project is in and on the required reliability with which the budget or the planned duration (quantification of the estimate or time-planning plus margin) will not be exceeded.

Considering the first point (size of the margin depends on the phase the project is in), in an early stage of the project one is much more uncertain about the budget estimate or the planned duration than in a later stage. Estimates and time-plans are often classified according to the phase of the project they were made in. Characteristic numbers for the magnitude and for the spreading around costs items depend on the project phase. For time-planning such characteristic numbers are not (yet) available.

11.7.4 Classification of uncertainty
Uncertainty can be classified in three categories:
♦ uncertainty related to Normal events;
♦ uncertainty related to Special events;
♦ project uncertainty.

Uncertainty related to Normal events
Although the costs items in a Basic budget estimate of a project become increasingly clear in the course of time, and the estimate becomes more accurate, many causes of uncertainty will remain as long as the project is not finished. With the necessary changes made, this can be applied to time-planning. The degree of uncertainty can be classified as follows:

1. There is no cause of uncertainty. The item concerned is deterministic. This concerns costs items or activities that are known exactly in size or duration. If, for example, the contract settling the purchase of land has been signed, this amount of money is known exactly. An "activity" with deterministic duration is the tide. The duration (along the North Sea coasts) is "exactly" 12 hours 25 minutes.

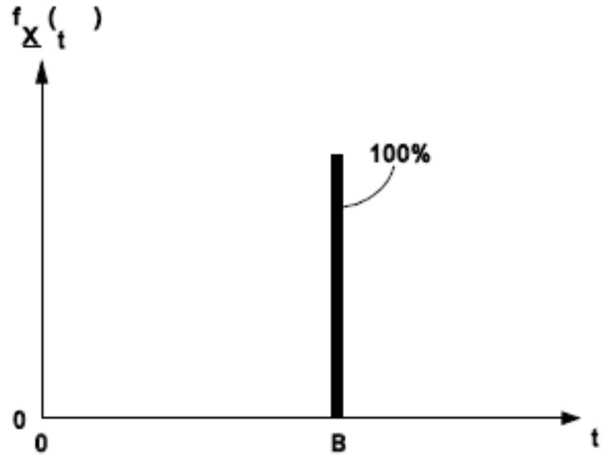

Figure 11-23: No cause of uncertainty

2. Often the costs are not so uniquely determined and one is uncertain about the duration of an activity. When the negotiations are still in progress, there is a notion about how much money the land (meant in point 1) will cost, but one cannot be certain. An example of uncertainty about the duration of an activity is a barge with heavy draught that has to be towed over a sill. Suppose this can only be done at high tide. (The keel clearance has to be sufficient). Usually the final costs or the spreading around the point in time of arrival at the sill will be within a band width. The probability density can then be as indicated in Figure 11-24.

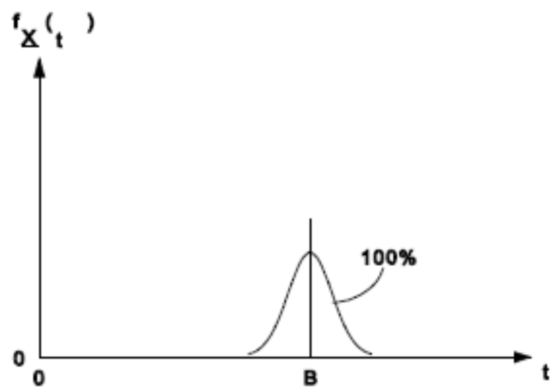

Figure 11-24: PDF of event B

Uncertainty related to Special events
Often another type of uncertainty plays a role with the evaluation of costs of a project or its duration, namely uncertainty caused by the Unforeseen or by Special events (mainly calamities). Two criteria characterize a Special event: the first is that it is not meant to occur and the second is that occurrence is not likely. The probability of occurrence, p, is small (less than 0.05), but if the event occurs, the consequence (damage or exceedance of the duration, B) is large. The probability of no occurrence (and accordingly: no damage or exceedance of the duration) is 1 - p. In a "classical" estimate of the budget or time-planning such events are seldom taken into account. Contractors insure against such events, associated with small probabilities but with large consequences. In a statistically controlled estimate or time-planning the probabilities and the consequences can be indicated as follows:

3. Figure 11-25 shows the mass density of a Special event as a function of the damage or exceedance of the duration.

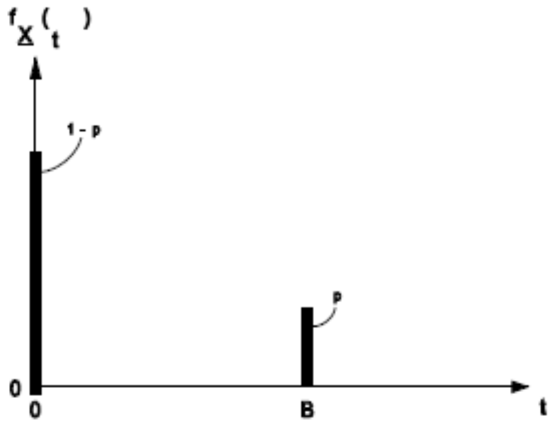

Figure 11-25: Mass density of a special event

4. The probability density function of a "Special event", of which the consequences (the damage or the duration) are subject to uncertainty, could be as is illustrated in Figure 11-26.

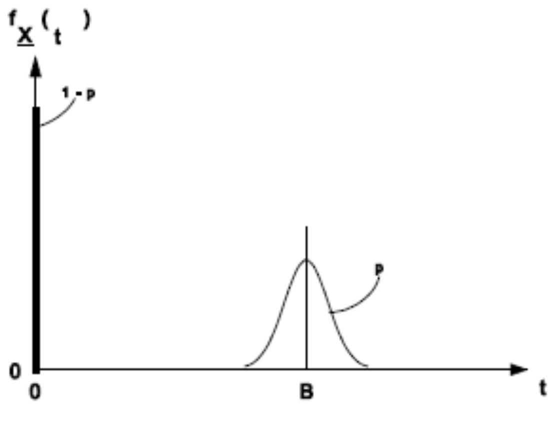

Figure 11-26: PDF of a special event

Project uncertainty or plan uncertainty: variants

5. In the study-of-plan phase several variants have to be considered and estimated (and sometimes planned). Beforehand, one is not certain which variant will be chosen (for example a tunnel, a ferry or a bridge for a road crossing of a river). Only at the end of the pre-design phase a decision is made. Awaiting the choice, elaborating and estimating (and eventually time-planning) several variants mainly meet the uncertainty. Sometimes, the decision between the variants is so unpredictable that all

variants are considered equally likely. Sometimes one variant is preferential and it is unlikely that another one will be chosen.

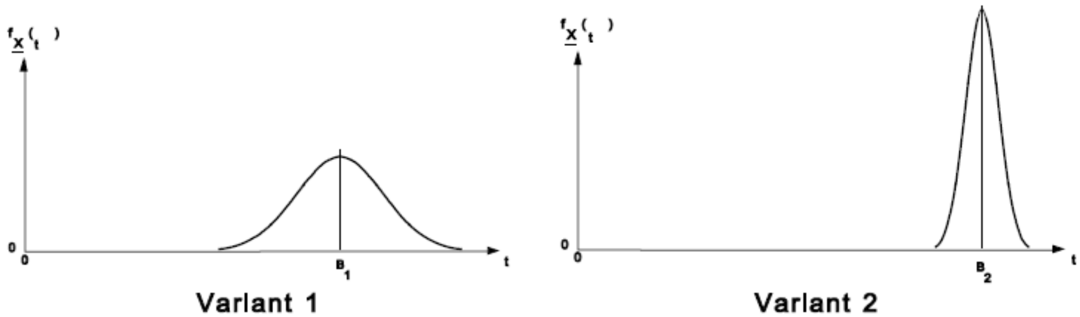

Figure 11-27: PDF of two example variants

If more than one variant is estimated or planned, the problem could be that the estimate of the budget is required to be one total amount of money or the time-planning should be one total duration. One estimate of the budget or one time-planning (possibly with a margin or expressed as a probability density function) for presentational purposes is then acquired rationally by weighing each variant by its (estimated) probability of selection. The following figure presents the result for two variants.

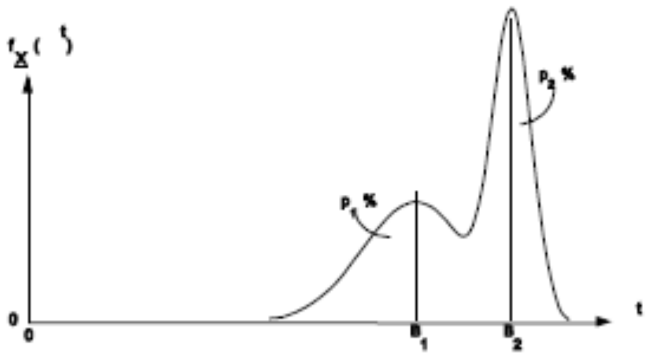

Figure 11-28: Result for two variants combined

The disadvantage is that the result is not recognized as a reasonable estimate or time-planning of each of the individual variants.

The classified uncertainty and the formulae to calculate the associated means and the standard deviations of the probability-weighed consequence (the related risks) are summarized in the following table. Mutatis mutandis the formulae hold for a time-planning with only one time-path.
Table 5:

	Case	mean	standard deviation	Description
Normal events	1	B	0	A deterministic amount of money, B, expressed in units of money
	2	B	$\sigma_{\underline{B}}$	A stochastic amount of money, with mean B, and some spreading σ_B
Special events	3	pxB	$\sqrt{p \times (1-p)}$	An event with probability of occurrence p that has a deterministic consequence B.
	4	pxB	$\sqrt{p((1-p)B^2 + \sigma_B{}^2}$	An event with probability of occurrence p that has a statistic consequence with mean B and some spreading expressed by σ_B
Plan uncertainty	5	p_1B_1 x p_2B_2	$\sqrt{p_1({B_1}^2 + {\sigma_{B1}}^2) + p_2({B_2}^2 + {\sigma_{B2}}^2) - (p_1B_1 + p_2B_2)^2}$	There are two (or more) variants, each associated with a probability of realization, p_i. Their probabilities add up to one as it is certain that one of the variants will be selected

The spreading for an item of the estimate increases with the related uncertainty. In the first case in Table 5, one is absolutely certain about the size of the sum, B. The standard deviation equals zero. In the second case there is some uncertainty. The spreading, $\sigma_{\underline{B}}$, is smaller than the expected value, *B*. (If this were not so, the estimate of the item was of no significance. It then suggests that there is not the vaguest idea of the size of B.) In case of Special events (cases 3 and 4), one is not certain if there will be costs (damage) at all. The probability that there will be costs is p (p << 1). There is a larger probability (1 - p) that there will be no costs. In fact the risk[7] 1): (probability × consequence = p × B) is estimated. If the Special event occurs, the estimated amount of money (p × B) is not nearly enough to cover the costs (B). According to this, the third case is associated with a larger spread (in the order of $B \times \sqrt{p}$) than in the case of a Normal event. So the spreading for Special events is approximately $\frac{1}{\sqrt{p}}$ times the expected value $p \times B$.

11.7.5 Calculation formulae for probabilistic budgeting

Monte Carlo simulations can be easily used in budgeting and time planning problems, but many expressions in budgeting and time planning problems can also be derived analytically. Assume that X and Y are random variables (of for instance prices and amounts) with PDF's f and g respectively. The following four functional operators are often encountered in budgeting and time planning problems:

Z=X+Y, U=X-Y, V=XY and W=X/Y

Then the PDF's of Z, U, V and W are, respectively, given by:

$f_Z(Z) = \int f(X)g(Z-X)\, dX$
$f_U(U) = \int f(U+Y)g(Y)\, dY$
$f_V(V) = \int f(X)g(V/X)\, |X|^{-1}\, dX$

[7] *From a mathematical point of view the estimates of Normal events are estimates of the risks as well. The probability of occurrence of Normal events is 1 (or 100% certainty)*

$f_W(W) = \int f(XW)g(X)\ |x|\ dX$

In textbooks on statistics the following relation is proven:

$E(XY) = E(X)E(Y)$

$$Var(\sum_{i=1}^{n} a_i X_i) = \sum_{i=1}^{n} a_i^2 Var(X_i) + 2\sum_{i=1}^{n}\sum_{j=i+1}^{n} a_i a_j Cov(X_i, X_j)$$

Furthermore it is possible to derive the following property for the product of random variables:

$Var(V) = Var(X)Var(Y) + E^2(X)Var(Y) + E^2(Y)Var(X)$

If exact calculations are not possible, the following approximation rules can be used (using Taylor's formula):

$$g(X) = g(m_X) + (X - m_X)\frac{dg(x)}{dx}_{|x=m_X} + \frac{(X-m_X)^2}{2}\frac{d^2g(x)}{dx^2}_{|x=m_X} + \ldots$$

From this, we can derive:
$E(g(X))\ .\ g(E(X))$

$Var(g(X))\ .\ Var(X)\ [g'(m_X)]^2$

If the coefficient of variation of X is less than c, the error involved in these approximations is less than c2. In particular, the following useful approximations in budgeting and time planning models can be used:

$$E(\sqrt{X}) \simeq \sqrt{E(X)}, \qquad Var(\sqrt{X}) \simeq \frac{Var(X)}{4E(X)}$$
$$E(X^{-1}) \simeq \frac{1}{E(X)}, \qquad Var(X^{-1}) \simeq \frac{Var(X)}{E^4(X)}$$

11.7.6 Dependent Bernoulli distributed random variables for special events
The notation X ~ BBG(p1,G1) is used for a random variable X which has a probability p1 of the occurrence of an event with consequences G1. Mean and standard deviation of X are:

μ=p1*G1
σ=G1*√(p1(1-p1))

Extensions to this univariate random variable by allowing 'horizontal and vertical uncertainties' in p1 and G1 are described by Van Gelder [10].

Dependence
The classical product-moment correlation coefficient of Pearson is usually used as a measure for dependence between two random variables. This correlation coefficient however, can only be applied for normal distributed random variables. For non-normal distributions, the correlation structure should be described differently. In the remainder of this section, a suggestion for such structure is proposed.

Bivariate BBG's
The following 4 situations can be distinguished:
1. X and Y are independent
2. X and Y 100% positively dependent
3. X and Y 100% negatively dependent
4. X and Y partially dependent

Situation 1
Assume X ~ BBG(p1,G1) and Y ~ BBG(p2,G2) and independence, then the following probability table can be derived:

Table 6.

X \ Y	0	G2	Sum
0	(1-p1)(1-p2)	p2(1-p1)	1-p1
G1	p1(1-p2)	p1p2	p1
Sum	1-p2	p2	1

Situation 2
If X and Y are completely positive dependent, then it follows,

- if X=0 then Y=0
- if X=G1 then Y=G2.

Using conditional probabilities: P(Y=0|X=0) = 1 and P(Y=G2|X=G1) = 1.

From which it follows: P(X=0 and Y=0) = P(X=0)*P(Y=0|X=0)=1-p1

and

P(X=G1 and Y=G2) = P(X=G1)*P(Y=G2|X=G1)=p1

and

P(X=0 and Y=G2) = P(X=0)*P(Y=G2|X=0)=0

and

P(X=G1 and Y=0) = P(X=G1)*P(Y=0|X=G1)=1-p1.

Which can be summarized in the following probability table:

Table 7.

X \ Y	0	G2	Sum
0	1-p1	0	1-p1
G1	0	p1	p1
Som	1-p2	p2	1

From this table, it follows that 1-p1=1-p2.

Therefore, BBG distributed X and Y can only be 100% positively correlated if p1=p2.

Situation 3
If X and Y are completely negatively dependent, then it follows that:

- if X=0 then Y=G2
- if X=G1 then Y=0.

The following probability table can be derived:
Table 9.

X \ Y	0	G2	Som
0	0	1-p1	1-p1
G1	p1	0	p1
Som	1-p2	p2	1

Requirement: p1=1-p2.

BBG distributed X and Y can only be 100% negatively correlated if p1+p2=1.

Situation 4
X and Y are partially dependent from each other:

- if X=0 then Y=0 with probability a1 and Y=G2 with probability 1-a1
- if X=G1 then Y=0 with probability b1 and Y=G2 with probability 1-b1.

Table 10.

X \ Y	0	G2	Sum
0	(1-p1)a1	(1-p1)(1-a1)	1-p1
G1	p1b1	p1(1-b1)	p1
Sum	1-p2	p2	1

Requirement: (1-p1)a1+p1b1=1-p2

BBG distributed X and Y can only be partially correlated if:

a1+p1(b1-a1)=1-p2.

Situation 4 is the most general situation. Situations 1, 2 and 3 can be derived directly from situation 4 with the following choices for a1 and b1:
Table 11.

	Choice for a1 and b1
Situation 1 (independencd)	a1=1-p2, b1=1-p2
Situation 2 (100% positively dependent)	a1=1, b1=0
Situation 3 (100% negatively dependent)	a1=0, b1=1

We conclude that the correlation structure between 2 BBG distributed X ~ BBG(p1,G1) and Y~BBG(p2,G2), needs to be described by 2 constants a1 and b1 which satisfy the following 3 boundary conditions:

$0 \leq a1 \leq 1$
$0 \leq b1 \leq 1$
a1+p1(b1-a1)=1-p2.

Complete positive dependence is reached when $a1 \to 1$ and $b1 \to 0$ and $p1 \to p2$.
Complete negative dependence is reached when $a1 \to 0$ and $b1 \to 1$ and $p1 \to 1-p2$.

11.7.7 Conclusions
Budgeting and time planning should be handled by probabilistic methods in order to deal with uncertainties prior to the start of the project. Correlations between events need special attention, in particular the correlation structure between special events. Analytical solutions are presented in this paper and a correlation structure is proposed.

11.7.8 Suggestions for further reading

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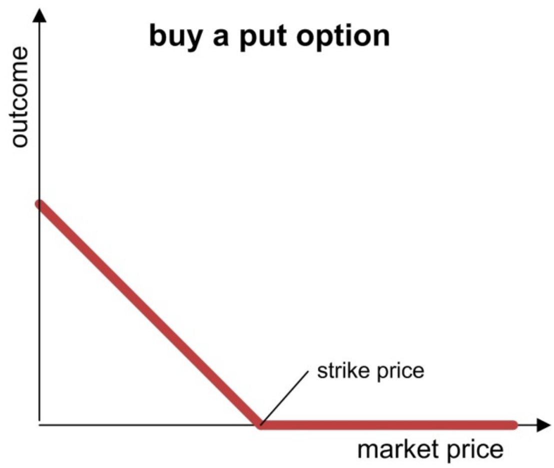
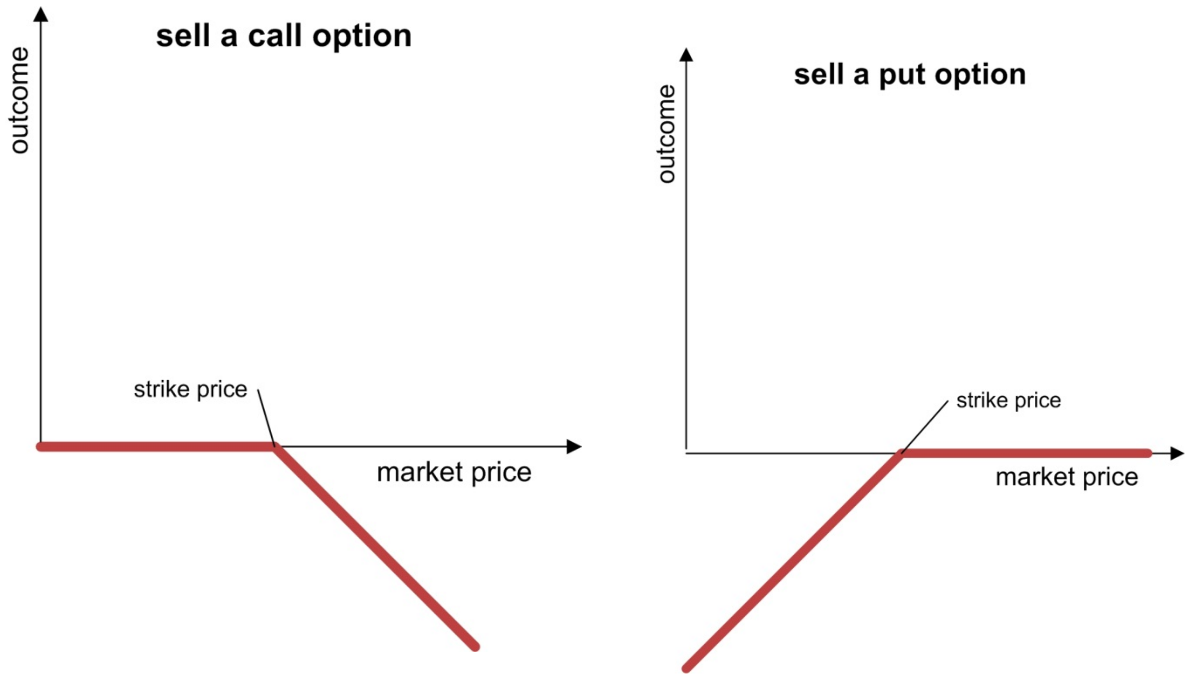
12 Options

An **option** is a contract between two parties concerning the buying or selling of an asset at a reference price during a specified time frame. It gives the owner of an option the *right* to buy or sell stock at a specified price on (or before) a specified day. This means an option does not contain any assets. An option can be interesting, because it fixes the price for which the asset will be sold/bought, while in the meantime the market price of this asset will change. This means an option creates a certainty or, in other words, reduces risk. Although an option contains no asset, it does create value for the buyer (this reduction of risk). This value is purely based on the expected future price movements of the asset it is linked to (the *underlying* asset). That's why such financial instruments are called derivatives.

So, a meatpacking company could take an option to buy live cattle at a fixed price. The option is the agreement that the cattle farmer will sell the live cattle at this fixed price. The option is *only* an agreement. The underlying asset is the live cattle. The option does not include the payment of these cattle, only an agreement about the price. Having this option is valuable for the meatpacking company, as they have now more certainty about the price of their resources.
In the meantime, the market value of live cattle changes continuously. The meatpacking company can decide on the moment the option matures, whether he will use the option or he will buy cattle at market price (which is only interesting when the market price is lower than the fixed price of the option).

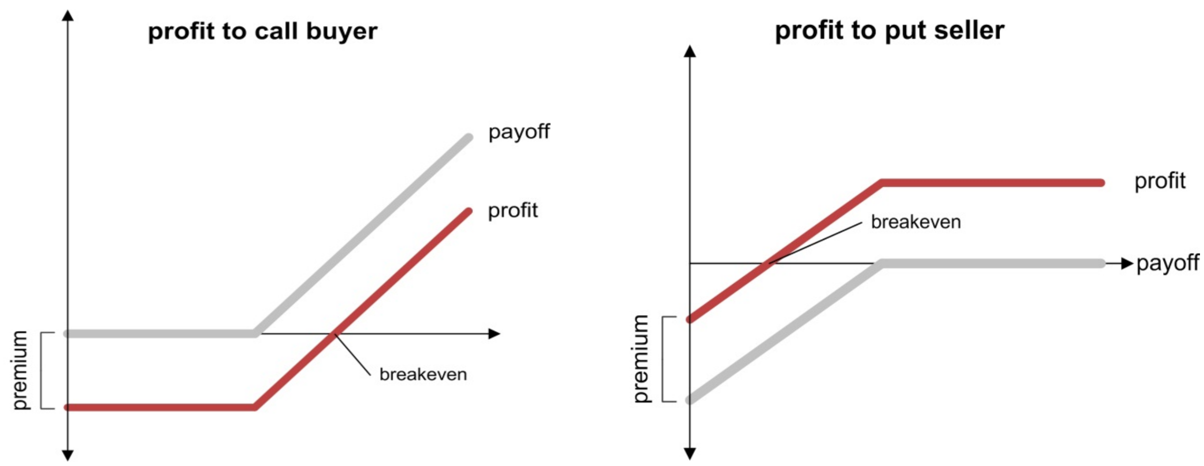
As explained, there are options for selling and options for buying: a **call option** gives its owner the right to *buy* stock at a fixed price, while a **put option** gives the right to *sell* a share at a fixed price. This fixed price is called a **strike** or **exercise price**. A call option is only interesting when the strike price lies below the market price and a put option only when the strike price lies above the market price. If not, the asset will be sold at market price (the option buyer can decide whether or not to use the option, the option seller has to do as he is told). This is shown in the following figures.





If you sell, or 'write', a call, you promise to deliver shares if asked to do so. In other words, the buyer's asset is the seller's liability.
If you call an option, you get the right to buy a share for a specified exercise price; the comparable put gives you the right to sell a share.

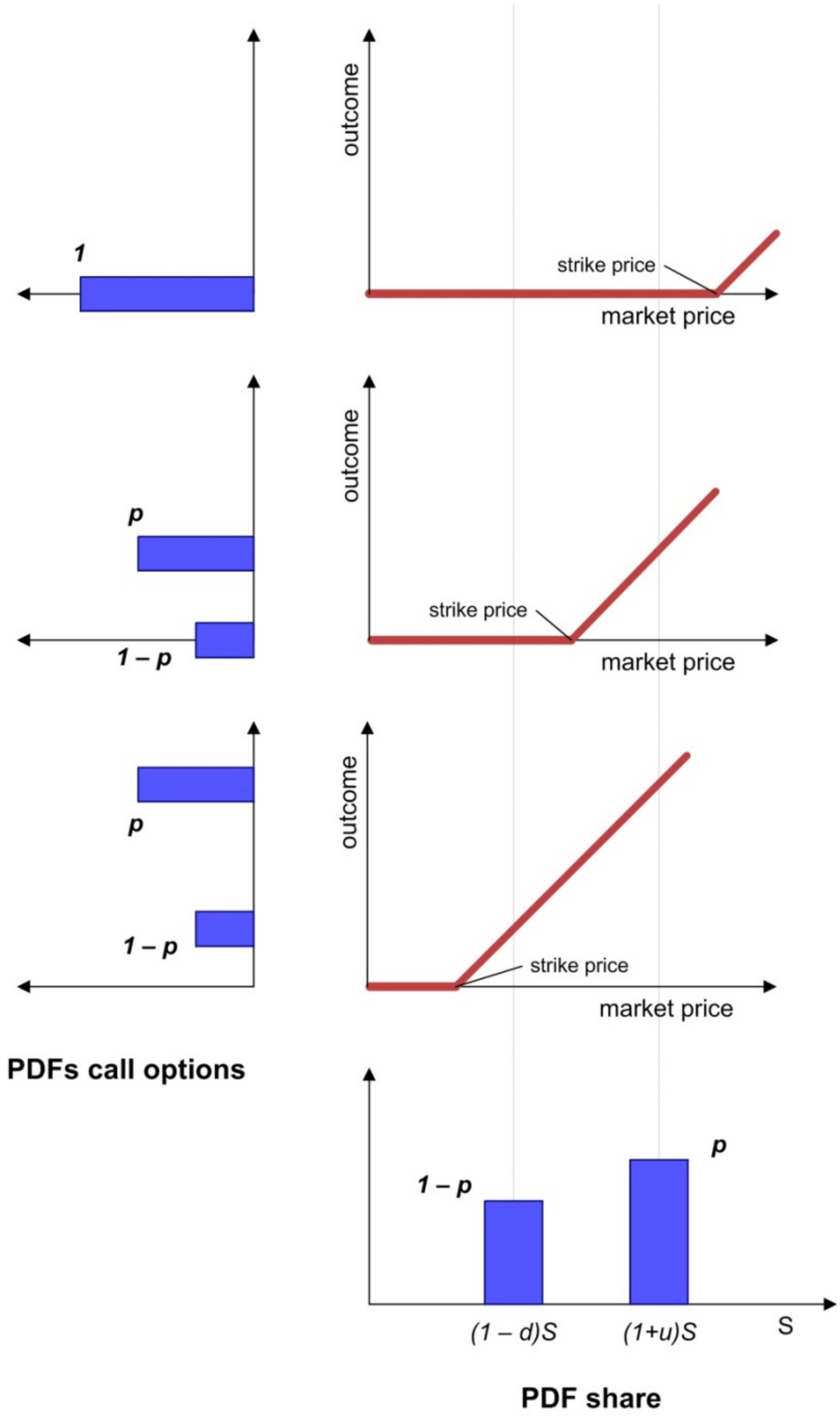
An option has a certain value and therefore a premium has to be paid to own an option. This results in the fact that payoff and profit differ as can be seen in the following figures.



So, the value of an option depends on the value of the underlying share. To determine the value of the option (remember: this value fully depends on the expected price movements of the underlying asset) we need to use the PDF (probability density function) of the share. The following figure shows what the effect is of different strike prices for options of the same share.

This share has a simplified PDF that shows two options:

- Upside change: probability p with the result $(1+u)S$ (with S being the current value of the share)
- Downside change: probability $1 - p$ with the result $(1 - d)S$

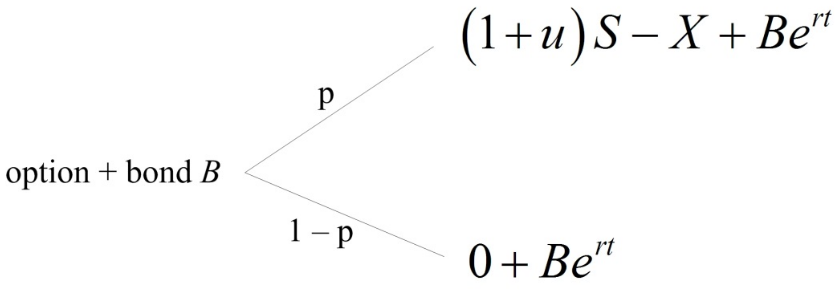

outcome
1
strike price
market price
outcome
p
strike price
1 – p
market price
outcome
p
1 – p
strike price
market price
PDFs call options
1 – p
p
(1 – d)S
(1+u)S
S
PDF share

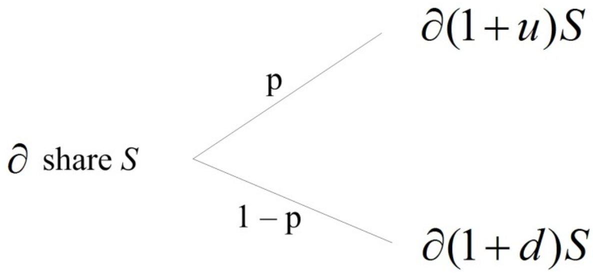
From these figures the following expected values for these call options (C) can be derived (with X being the strike price):

1. $E(C) = 0$
   $Var(C) = 0$
2. $E(C) = p\{(1+u)S - X\} + (1-p)\cdot 0$
   $Var(C) = p(1-p)\{(1+u)S - X\}^2$
3. $E(C) = p\{(1+u)S - X\} + (1-p)\{(1+d)S - X\}$
   $Var(c) = p(1-p)\{(1+u) - (1+d)\}^2 \cdot S^2$

By combining options and other financial products, one can create every payoff scheme (strategy). One can think of downside protection, meaning that the structure cannot payoff less than a certain amount. There can also be other strategic structures that can be useful for all kind of reasons, for example to compensate the result of some other financial arrangement.
The book describes two examples: imitating the buying of a put (without buying a put) [p.573] and imitating the bonus structure of Ms. Higden [p.575].

Arbitrage





As well upside as downside must be equal in value:

1. $(1+u)S - X + Be^{rt} \overset{m}{=} \partial(1+u)S$
2. $0 + Be^{rt} \overset{m}{=} \partial(1+d)S$

From these two equations:

$$\partial = \frac{(1+u)S - X}{\{(1+u)-(1+d)\}S}$$

$$B = \frac{\{(1+u)S - X\}(1+d)}{(1+u)-(1+d)} \cdot e^{-rt}$$

Up: $P = \partial(1+u)S = \frac{(1+u)S - X}{\{(1+u)-(1+d)\}S} \cdot (1+u)S$

Down: $P = \partial(1+d)S = \frac{(1+u)S - X}{\{(1+u)-(1+d)\}S} \cdot (1+d)S$

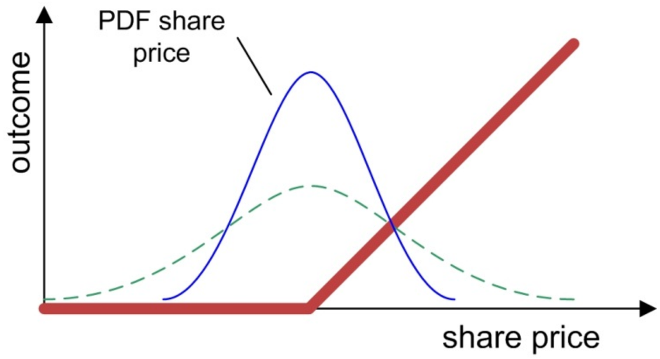
The investment at t_0 is equal:

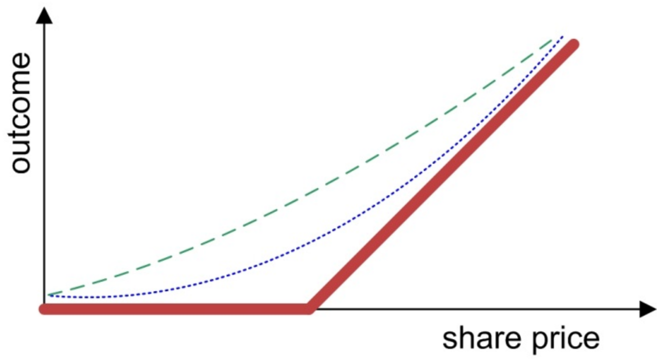
$$C + B \overset{m}{=} \partial S$$

$$\boxed{C \overset{m}{=} \partial S - B}$$

Value of the call option:

$$C = \left\{ e^{rt-(1+d)} \right\} \frac{(1+u)S - X}{(1+u)-(1+d)} \cdot e^{-rt}$$





value of call option

The relationship between the call and put prices are as follows:

$$Value\ of\ put = value\ of\ call + present\ value\ of\ exercise\ price - share\ price$$

Black-Scholes formula

$$\text{Value call option} = \partial \cdot S - B = N(d_1) \cdot S - N(d_2) \cdot PV(X)$$

$$d_1 = \frac{\log\left(\frac{-S}{(PV(X))}\right)}{\sigma\sqrt{t}} + \frac{\sigma\sqrt{t}}{2}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

$N(d) =$ cumulative normal distribution

$X =$ strike price

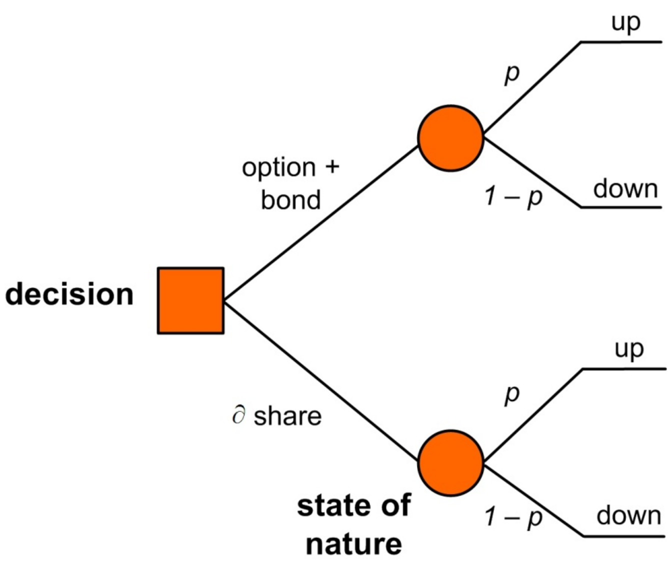
$S =$ share price now

$\sigma =$ standard deviation of rate of return

$PV =$ present value

Decision tree

Expected value of the decision



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